ECON 110, Prof. Hogendorn

Problem Set 5

1. *Returns.* Graph each of the following production functions and explain whether there are diminishing returns in each case? Consider a firm that has these production functions. What is the firm's conditional demand for labor? (Let w be the wage and p be the price of the product.)

   (a) $f(L) = 20L^{1/2}$

   (b) $f(L) = 16002L - L^2$

2. *Nineteen.* A firm's production function is $q = f(L) = 10 + L^{1/3}$. The wage of labor is \$10. The firm has a fixed cost of \$47,500.

   (a) What are this firm's total, marginal, average, and average variable cost curves? (Hint: as a general rule, don't expand expressions like $(a + b)^c$ unless you really have to!)

   (b) Suppose the firm is a perfect competitor and the price of the good is \$3,000. How much profit does the firm make? How much labor is employed?

   (c) If the price fell by 19%, what would be the percentage change in profits and employment at this firm? Graph what happens in two ways: on a graph of the marginal and average cost curves and on a graph of the production function.

   (d) After the price falls, should the firm shut down?

3. *GMToyota.* Let General Motors and Toyota have two small factories, each with exactly the same production function for producing cars:

$$f(L) = 316L^{1/4}$$

Each company makes a single type of car that sells for a price of $p = \$25,000$. Each worker's annual salary is $\$62,500$. Each company makes 1000 cars per year at its factory.

(a) What is the conditional factor demand for cars? What is the average variable cost and marginal cost of a car?

(b) Toyota has a fixed cost of $15,000,000 at its factory. What is its operating profit and its net profit? Show the profits on a graph of price, average cost and average variable cost.

(c) GM has the same $15,000,000 fixed cost, plus additional fixed costs of $6,000,000 due to pensions for retired employees. What is its operating profit and its net profit? Show the profits on a graph of price, average cost, and average variable cost.

(d) Assume production is fixed at 1000 cars and does not change from year to year. Toyota's factory will last for 5 years. Car prices and workers' salaries are both projected to grow at 5% per year. The production function will not change, and the same $15 million fixed cost occurs every year. The factory will have no value at all after 5 years. If the interest rate is 10%, how much is the factory worth today?

4. *USAirways.* In fall 2006, US Airways announced a hostile takeover bid for Delta Air Lines. The investment community and the airline industry responded very positively. Everyone thought that a merger like this would reduce competition and raise profits for *all* airlines, not just the two merging firms.

(a) With reference to the formula for pricing a stock, did airline stocks rise or fall after the announcement?

(b) With reference to the formula for pricing a bond, did airline bond yields rise or fall? Did airline bond prices rise or fall? Why?

(c) Did the news change the coupon payments on airline bonds with a $100 face value issued in August 2006? Is it likely to have changed the coupon payments on airline bonds with a $100 face value issued in December 2006 (i.e. bonds not yet issued at the time of the announcement)? Why?

Review Problems only, not to turn in:

5. *Low.* Suppose a firm has cost curves $MC(q) = 0.0512q$ and $AC(q) = \frac{50}{q} + 0.0256q$. Use the first derivative of AC to prove that MC crosses AC at the lowest point on the AC curve.

6. *EightFirms.* Suppose there are 8 firms supplying a given market. Each firm has the same total cost curve, which is

$$TC(q) = 20 + 12q + 2q^2$$

Each of the firms is a perfect competitor. *Market* demand is $q(p) = 60 - p$. What is the equilibrium price in this market? How much does each firm produce? Draw graphs to illustrate your answer.

7. *Long.* Derive and graph the long-run competitive equilibrium price associated with the following long-run total cost curve: $TC(q) = 1000 + 50q^2$.

8. *Lula.* Suppose there is a Brazilian government bond with a face value of R$100 (i.e. 100 reals, the currency of Brazil). The bond has a coupon of R$5 and matures in 1 year.

   (a) If the bond's current price is R$80, what is its yield?

   (b) Many investors thought that if Lula da Silva were elected president of Brazil, Brazilian debt would become more risky. Explain what probably happened to the price of Brazilian government bonds when Lula won.

9. *Jetta.* Suppose you were thinking about buying (with a car loan) or leasing a Volkswagen Jetta.

   Buying with loan: To make this simple, let's assume that each year you pay your entire car payment at the end of the year. Based on Volkswagen's current financing offer, you would pay $6,186 at the end of year 1, 2, and 3, and then you would own the car. According to the web site, the car will be worth $10,739 at that point.

   Leasing: Volkswagen has an offer where you can lease the car with no up-front costs. Again let's make the entire lease payment due at the end of the year, in which case the lease payments are $3,237 at the end of year 1, 2, and 3. After that you just have to give the car back to Volkswagen.

   Let's use 5% as the interest rate for this problem.

   (a) Write down the formula for the present value of the lease offer, and then find the present value.

   (b) Write down the formula for the present value of the buy with loan offer, and then find the present value.

   (c) Suppose that you take the buy offer, but then at the end of year 1, right *after* you make the $6,186 payment, you decide to sell the car. The person you sell to will have to pay the remaining two payments. That person gets a value equal to $3,237 in each of the two years from having a car to drive. How much will the buyer pay?

Answers to Review Problems:

5. *Low_a.* At the lowest point on the AC curve, the slope is 0:

$$\frac{dAC}{dq} = -\frac{50}{q^2} + 0.0256 = 0 \Rightarrow q^2 = 1953.125 \Rightarrow q = 44.2$$

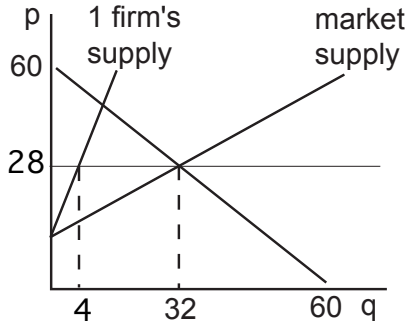
Setting MC=AC gives us

$$\frac{50}{q} + 0.0256q = 0.0512q \Rightarrow \frac{50}{q} = 0.0256q \Rightarrow q^2 = 1953.125 \Rightarrow q = 44.2$$

Either method gives the same answer.

6. *EightFirms.* Each firm has marginal cost curve $MC(q) = 12 + 4q$. Since each firm will optimally set price equal to marginal cost, we invert this curve and each firm has supply curve $s_i(p) = \frac{1}{4}p - 3$. Then market supply is eight times this, or $s(p) = 2p - 24$.

Market equilibrium occurs where supply equals demand, or $2p - 24 = 60 - p \Rightarrow 3p = 84 \Rightarrow p = 28$. Each firm produces $s_i(28) = \frac{1}{4}28 - 3 = 4$.



7. *Long_a.* In the long run, there will be entry if $p > AC$ and exit if $p < AC$. Therefore we are looking for a point where both $p = MC$ (short-run optimizing) and $p = AC$ (long-run equilibrium). The only such

point is where:

$$\begin{aligned} MC(q) &= AC(q) \\ 100q &= \frac{1000}{q} + 50q \\ 50q &= \frac{1000}{q} \\ q^2 &= 20 \\ q &= 4.47 \\ p = MC(4.47) &= 447 \end{aligned}$$

8. *Lula.*

   (a) The formula to use here is

   $$P = \frac{A}{1+i} \quad R\$80 = \frac{R\$5 + R\$100}{1+i} \Rightarrow 1 + i = 1.3125 \Rightarrow i = 31.25\%$$

   (b) Investors perceived Lula as risky, and they demanded a higher risk premium on Brazilian government bonds. For an existing bond, the coupon and face value have already been set, so the only way for the yield to rise was for the present value to fall, as shown in the formula above.

9. *Jetta_a.*

   (a) This formula is quite simple:

   $$\frac{-3237}{1.05} + \frac{-3237}{1.05^2} + \frac{-3237}{1.05^3} = -8815$$

   (b) Here the formula is a little more complex because you have the residual value of the car after the loan is paid off:

   $$\frac{-6186}{1.05} + \frac{-6186}{1.05^2} + \frac{-6186}{1.05^3} + \frac{10739}{1.05^3} = -7569$$

(c) Here there are several interesting issues. First, we are talking about a sale that will take place at the end of year 1. Although that is 1 year in the future, the question asks for the amount that will be paid *at the time of the sale,* so the amount should not be discounted back to the present.

Second, the buyer will not end up paying very much for the car because they are also assuming the loan.

Third, the buyer ends up with the residual value of the car after the loan is paid off. The formula is thus:

$$\frac{-6186 + 3272}{1.05} + \frac{-6186 + 3272}{1.05^2} + \frac{10739}{1.05^2} = 4322$$