

ECON 110, Prof. Hogendorn

Problem Set 5 Answers

1. *Returns_a.*

- (a) The graph is an increasing, concave function. $MP_L = 10L^{-1/2}$ which falls in L so there are diminishing returns. To find conditional labor demand,

$$q = 20L^{1/2} \Rightarrow L^{1/2} = \frac{q}{20} \Rightarrow L(q) = \frac{q^2}{400}$$

- (b) This is also increasing concave, but it eventually reaches a maximum and then declines thereafter. This occurs at very large amounts of labor, however. $MP_L = 16002 - 2L$ also falls in L so there are diminishing returns. To find conditional labor demand,

$$q = 16002L - L^2 \Rightarrow (-1)L^2 + 16002L + (-q) = 0$$

Using the quadratic formula,

$$L(q) = \frac{-16002 \pm \sqrt{4 \cdot (-1) \cdot (-q)}}{2 \cdot 16002}$$

Only the positive root makes any sense, so it must be

$$L(q) = -\frac{1}{2} + \frac{\sqrt{q}}{16002}$$

2. *Nineteen_a.*

- (a) Inverting the cost function gives $L = (q - 10)^3$. Then the cost functions are:

$$TC(q) = 47500 + wL = 47500 + 10(q - 10)^3$$

$$\begin{aligned}
MC(q) &= 30(q - 10)^2 \\
AC(q) &= \frac{47500}{q} + 10\frac{(q - 10)^3}{q} \\
AVC(q) &= 10\frac{(q - 10)^3}{q}
\end{aligned}$$

(b) Write the profit function as $\pi(L)$ instead of $\pi(q)$:

$$\max_L \pi(L) = pq - TC(q) = 3000(10 + L^{1/3}) - 47500 - 10L$$

Then the first order condition is:

$$\frac{d\pi}{dL} = 1000L^{-2/3} - 10 = 0 \Rightarrow L = 1000$$

$$\text{Profit is } \pi(1000) = 3000(10 + 10) - 47500 - 10000 = 2500.$$

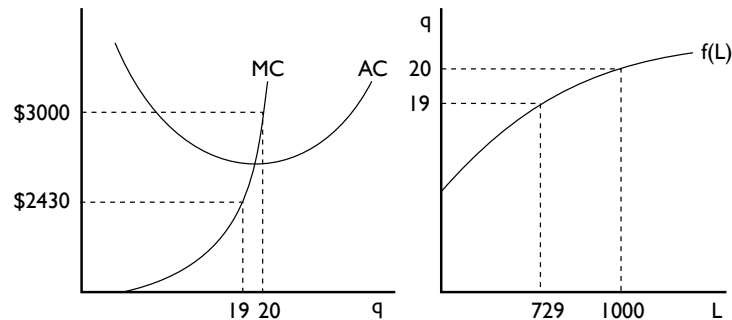
(c) The new first order condition would be

$$\frac{d\pi}{dL} = (1 - 0.19)1000L^{-2/3} - 10 = 0 \Rightarrow L = 729$$

The new profit is

$$\pi(729) = (1 - .19)3000(10 + 9) - 47500 - 7290 = -8620$$

Thus, employment falls by 27.1% and profit falls by a whopping 445%!



(d) The new quantity is $f(729) = 19$, and $AVC(19) = 10\frac{(9)^3}{19} = 383.7$. This is less than the new price of $(1 - 0.19)3000 = 2430$. The firm should not shut down because it more than covers its variable costs, and in fact makes quite a large contribution to fixed costs. In the long run, however, it should shut down.

3. *GMToyota_a*. Let General Motors and Toyota have two small factories, each with exactly the same production function for producing cars:

$$f(L) = 316L^{1/4}$$

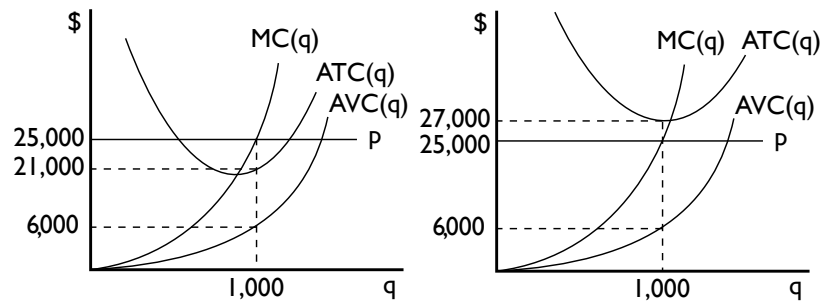
Each company makes a single type of car that sells for a price of $p = \$25,000$. Each worker's annual salary is $\$62,500$. Each company makes 1000 cars per year at its factory.

- (a) By inverting the production function, we find that $L(q) = \left(\frac{q}{316}\right)^4$. Then total variable cost is $TVC(q) = 0.000006q^4$, average variable cost is $AVC(q) = 0.000006q^3$, and marginal cost is the derivative of total variable cost, or $MC(q) = 0.000025q^3$.
- (b) Operating profit is

$$\pi_T = (p - AVC(1000))1000 = (25000 - 6000)1000 = 19,000,000$$

$$\text{Net profit is } \Pi_T = \pi_T - F = 19,000,000 - 15,000,000 = 4,000,000.$$

The graph looks something like the left panel below:



- (c) GM has precisely the same variable costs, so it also has the same marginal cost, average variable cost, and operating profit. The only difference is the net profit, which is exactly $\$6,000,000$ less, or $\Pi_G = -2,000,000$. The graph is like the right panel above.
- (d) The operating profit will grow by 5% each year, since both its components, revenue and variable cost, grow by 5%. Thus, the

discounted present value formula is:

$$\frac{1.05\pi_T - F}{1.10} + \frac{1.05^2\pi_T - F}{1.10^2} + \frac{1.05^3\pi_T - F}{1.10^3} + \frac{1.05^4\pi_T - F}{1.10^4} + \frac{1.05^5\pi_T - F}{1.10^5}$$

Looking at this equation, it makes more sense to evaluate the two separately, because this is equivalent to the PV of the fixed cost discounted at 10% and the PV of the *current* operating profit discounted at only 5%. These are

$$PV = \pi_T (1.05^{-1} + 1.05^{-2} + 1.05^{-3} + 1.05^{-4} + 1.05^{-5}) = 82,260,057$$

and for the fixed cost

$$PV = F (1.10^{-1} + 1.10^{-2} + 1.10^{-3} + 1.10^{-4} + 1.10^{-5}) = 56,861,801$$

Thus, the total value of the factory is $82,260,057 - 56,861,801 = 25,398,256$.

4. *USAirways_a.*

- (a) The price of a stock is the discounted present value of the future profits of the company (divided by the number of shares of stock outstanding). So if π_t is the profit per share in year t , the price of one share of stock should be

$$p = \frac{\pi_1}{1+r} + \frac{\pi_2}{(1+r)^2} + \frac{\pi_3}{(1+r)^3} + \dots$$

where r is the appropriate interest rate for discounting the future. The merger proposal made investors think that the profits will rise, so the price of the stock will rise. It also made the stocks less risky, so the interest rate for discounting does not need to be as high as before.

- (b) The price of a bond is the discounted present value of the stream of coupon payments, plus the face value that is returned at the

maturity date. So if c is the coupon payment, f the face value, and m the maturity date, the price is

$$p = \frac{c}{1+r} + \frac{c}{(1+r)^2} + \dots + \frac{c}{(1+r)^m} + \frac{f}{(1+r)^m}$$

where r is the interest rate used for discounting, also called the yield of the bond. The merger proposal decreased the risk of holding these bonds (particularly for the airlines in bankruptcy), so the yield decreased. Since the yield is in the denominators, the price increased.

- (c) Bonds are a type of financial contract, so nothing about the news can change the terms of the contract such as the coupon payments or the maturity date of a bond that was already issued. However, bonds to be issued in the future will be priced according to the required yield at the time they are issued. The merger announcement reduced the risk, and therefore the required yield, of airline bonds. Therefore, the new airline bonds will be priced with lower coupon payments reflecting a lower required return on the face value.