ECON 110, Prof. Hogendorn

Problem Set 6

1. *LaborMarket.* Suppose that all firms in the economy have the production function $f(L) = 20L^{1/2}$, and there are 1000 firms. Let L measure hours of labor. Suppose there are 100,000 workers, and each one has a vertical labor supply curve of 8 hours per day. If $p = 10$, what is the equilibrium wage?

2. *Generators.* It is 2 in the afternoon on a hot day in July. Everyone in the city has their air conditioning turned up, with the result that the typical household demands 6 kWh (kilowatt hours) of electricity during the 2-3pm time slot. Their demand is perfectly inelastic because they are so hot, they don't care about the price of electricity!

   There are two generating companies (GenCos) serving this city. Each one operates an oil-fired power plant that can produce electricity according to the production function

   $$f(g) = (540g)^{\frac{1}{3}}$$

   where g is gallons of oil and q is kWh of electricity per household in the city. The price of oil is 200¢ per gallon. Each GenCo also has a fixed cost of 20¢ per household. The price of electricity is p and the GenCos are price-takers.

   (a) The managers at GenCo A like to maximize their profits in terms of the quantity $q$of electricity per household that they produce. Write down the GenCo A profit function $\Pi(q)$. Derive GenCo A's supply curve.

   (b) The managers at GenCo B work differently. They figure out how much oil to buy to maximize profits. Write down the GenCo B

profit function $\Pi(g)$. Derive GenCo B's supply curve (this will require an extra step relative to your answer for part a).

(c) Describe in words why the two GenCos end up with the same supply curves.

(d) What is the market equilibrium price of electricity? Draw a graph of the market equilibrium.

(e) Derive and graph the marginal and average cost curves for one of the firms. At the market equilibrium price, calculate and label on the graph the profit or loss of the firm.

(f) Describe what will happen in this market in the long run, and show the effects in both the market graph and the graph of an individual firm. Just show how the curves will shift; don't calculate the actual quantities and prices in the long run.

3. *Boomerangs.* Amherst Guy was fired from Barn & Company for his ridiculous advice to Dolty. Now he is managing a boomerang factory outside of Perth. The factory has a production function $q = f(v)$, where v is wood and q is boomerangs. The company is a price-taker in both the boomerang and wood markets.

Amherst Guy says to the company's president, "You know I went to Amherst, so I actually know TWO ways to maximize profits. I could choose the optimal quantity of boomerangs by setting marginal cost equal to average cost, or I could choose the optimal amount of wood by setting the price of wood equal to the unemployment rate. Either way, I will get the same supply curve for our firm."

(a) Correct AG's statement.

(b) AG wanted to set marginal cost equal to average cost for his first method. Is there anything special about that point? Is it likely that the firm will end up doing this? Illustrate with a graph.

(c) Suppose the Australian government decides that boomerang production causes a negative externality due to deforestation. Suppose that it decides to correct this externality using a Pigouvian tax. Show what will happen on a supply and demand diagram for the boomerang market. Show any deadweight losses created or avoided by the tax.

(d) If the explicit functional form of the production function is $q = 5v^{1/3}$, what is the boomerang maker's conditional demand for wood and what is its unconditional demand for wood? (Assume p is the price of boomerangs and p_v is the price of wood.)

4. *SmallCountry.* Remember that a country's supply of loanable funds is the *net* supply after households that borrow are subtracted from those who save. Suppose there is a small country with 1000 households. 700 of these have a savings function $s = 50r$, where r is the rate of return on capital. The remaining 300 households have savings function $s = -1 + 10r$. (You can imagine that both the number of households and the amount of savings are in thousands.)

   (a) Graph the individual and aggregate savings functions. Describe in words what happens to both types of household and the whole country when the interest rate rises from 3% to 11%.

   (b) There are 100 firms, and each firm has an investment demand function $(i(r) = 10/r$. Find and graph the aggregate investment function for the whole country.

   (c) Show that the equilibrium interest rate in this country is 16.6% (rounded to one decimal).

   (d) In most countries, a real interest rate of 7% would be more typical. Do you think this country will have higher or lower economic growth than the typical country? Explain.

Review Problems only, not to turn in:

5. *Nokia.* According to the *Wall Street Journal,* Nokia produces its cellphones using a very different production function than its competitors:

> David Pringle, ``How Nokia Thrives by Breaking the Rules,'' January 3, 2003, pg. A7.
>
> Salo, Finland -- THIS TOWN of 25,000 people on the northern fringe of Europe may not seem the obvious choice for a mobile-phone factory. Accessible only via a two-lane highway, Salo is a 90-minute drive from Helsinki's international airport. Average wages in the region are around 30 times those in low-cost manufacturing centers, such as China.
>
> Yet the Nokia Corp. plant in Salo is the nerve center of the mobile-phone company's global-manufacturing operation. Rather than joining Dell Computer Corp., Cisco Systems Inc. and other big technology firms in contracting out manufacturing to low-cost specialists in Asia, Nokia makes most of its phones in its own factories, some of them in expensive locations, such as the U.S., Germany and this town in southern Finland.
>
> Goldman Sachs estimates Nokia is making low-end handsets for as little as $70, matching the best of the Asian makers.
>
> Nokia's operating margin on phones was 22% in the third quarter, and the company promised in December to top that in the fourth quarter.

Suppose the the price of a low-end cellphone is $85.40, the wage in China is $1 per hour and the wage in Finland is $30 per hour. We assume that workers in China and Finland are identical in terms of their

own personal skills, trainability, etc.

(a) Suppose that Nokia's factory in Finland employs 650 workers and that a cellphone factory in China employs 10,000 workers. Both factories produce the 1,516 phones per hour. On the same graph, draw production functions $q(L)$ for both of these factories (simply draw graphs, you don't have enough information to know the exact mathematical form of the production function). Assuming the firms are profit maximizers, what is the marginal product of labor at the Chinese firm? At Nokia's Finland plant?

(b) Which firm's labor demand curve is further to the right on a graph (with L on the x-axis and w on the y-axis)? Why?

(c) True or false: The fact that a *business* newspaper like the *Wall Street Journal* says that Nokia makes a profit margin of 22% shows that we cannot use the perfect competition model to think about Nokia because it does not set price equal to marginal cost. Draw a well-labeled cost-curve graph to justifies your answer.

6. *MBAs.* The last recession was very hard on the strategic consulting industry. Firms like McKinsey, Bain, and Booz Allen & Hamilton laid off 30% of their workforce.

   There were two components to the downturn. First, demand fell dramatically, in large part because of the demise of the dot-coms. Second, more executives began to have business school degrees and/or experience with the consulting firms. This made the ``sage advice'' of the consultants themselves less useful and effectively reduced the marginal product of laborers with MBA (Master of Business Administration) degrees (see *The Economist*, 11/2/02, pg. 61).

   For this problem, assume that the wage of MBAs is $100. (Note: for more realism, you can think of all money amounts in this problem in thousands.)

(a) Let a typical consulting firm have production function $f(L) = 10000L^{1/2}$ and the firm also incurs a fixed cost of 1000. What is this firm's total cost function, average cost function, average variable cost function, and marginal cost function?

(b) Graph these curves.

(c) If the price of consulting is $p = 2$ and there are 5 consulting firms, how many MBAs are hired?

(d) Suppose that p falls to 1.60 and also the production function changes to $f(L) = 10000L^{149/300}$. Now how many MBAs are hired?

7. *Revolution.* After Wesleyan, you take a job with McCoy consulting. It was a tough decision because McCoy's big rival, Barn & Co., was also recruiting you. And now the pressure is on because you are making a big presentation to Dolty, an auto parts manufacturer which is a perfectly competitive firm.

   (a) The perfectly competitive price of a car bumper is \$500. Dolty uses steel to make bumpers according to the production function $f(S) = 1000S^{\frac{1}{12}}$ where S is tons of steel. The price of steel is \$800 per ton. What is Dolty's profit function $\Pi(q)$?

   (b) Describe the condition for profit maximization that shows how many bumpers Dolty should produce.

   (c) After you have shown the above, a team from Barn & Co. bursts into the room. Their young leader, known only by his initials A.G., says "Barn has a revolutionary new way to manage your firm. Don't think about bumpers, like these dinosaurs from McCoy! Instead, decide how much steel to buy!" He proceeds to write down a profit function $\Pi(S)$. Assuming he does this correctly, what does he write down? Show the condition for profit maximization using this function.

(d) Now it's up to you to save McCoy's reputation. Argue (in words) that the profit maximization condition for A.G.'s method is exactly the same as the profit maximization condition in your method, and that Barn & Co. has no revolutionary management technique.

8. *UncleKarl.* Your Uncle Karl gives you 20,000 dollars of capital.

   (a) For $1000, you can buy a risk-free government bond with a coupon of $50 (payable at the end of the year), a face value of $1050, and a maturity of one year. What is the yield on this bond?

   (b) Alternatively, you can invest some of the capital in a business venture producing downloadable music. For each dollar of capital invested over the course of one year, do you think it is more reasonable to let your cost of that capital be $0.05, $0.10, or $0.15? Discuss your answer with reference to part (a), assuming you can buy fractional amounts of the bonds.

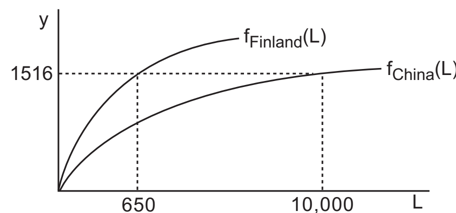
   (c) To simplify, assume no labor is involved in this business; the only factor is capital. Your production function is $q(K) = 100K^{9/10}$, where output is measured in the number of downloads. You must also use $5,000 more of capital to pay a fixed cost to get started. What are the equations for your total, average, and marginal cost curves, using your answer to (b)? Graph the AC and MC curves.

   (d) If each download brings you revenue of $0.04, how much capital should you invest in this business? Show this on your graph. Do you earn a competitive rate of return on your capital, or do you receive rents?

Answers to Review Problems:

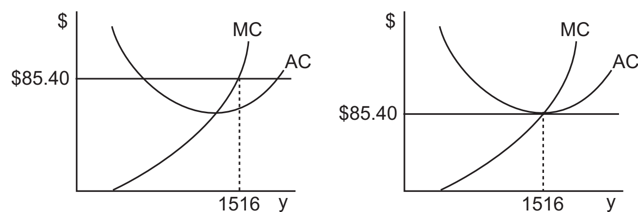
5. *Nokia_a.*

   (a) Any profit maximizing firm will set $pMP_L = w$, which can also be written $MP_L = w/p$. Here the price is \$85.40, so in Finland $MP_L = 0.351$ and in China it is $MP_L = 0.012$. You can see that the MP_L must be lower in China from thegraph of the production function.

   (b) For any given quantity of labor, the MP_L is higher in Finland. Therefore at any given wage, there is more demand for labor in Finland, and the Finnish labor demand curve is shifted further to the right. However, since the wage is much higher in Finland, the quantity of labor demanded is lower there.

   (c) There are two reasons that the newspaper statement is not a problem. First, the article does not imply that the market is in long run equilibrium. So the situation could very well be like the short-run graph shown on the left, where price equals marginal cost but is well above average cost.

   Second, even if the market is in long run equilibrium, as in the graph on the right, that just says that price equals average cost

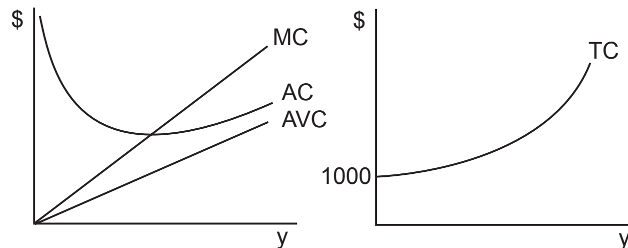
from an economist's point of view. The economist's version of costs includes the cost of capital. And in this industry, risk is very high and the capital depreciates very rapidly because of technological change making it obsolete. Therefore a cost of capital of 22% may be entirely justified.

6. *MBAs_a.*

   (a) Since $y = 10000L^{1/2}$, $L(y) = \left(\frac{y}{10000}\right)^2$. Thus,

$$\begin{aligned} TC(y) &= 1000 + wL = 1000 + 100\left(\frac{y}{10000}\right)^2 \\ AC(y) &= \frac{1000}{y} + \frac{y}{1000^2} \\ AVC(y) &= \frac{y}{1000^2} \\ MC(y) &= \frac{y}{500000} \end{aligned}$$

   (b)



   (c) We know that a profit-maximizing, perfectly competitive firm sets $p = MC(y)$. Here, that implies

$$\frac{y}{500000} = 2$$

   Solving this for y, we find that $y = 1,000,000$. Then $L(1,000,000) = 10,000$. Since there are 5 such firms, the total number hired is 50,000.

(d) Now the labor needed is:

$$L(y) = \left(\frac{y}{10000}\right)^{300/149}$$

and the optimal output solve:

$$MC(y) = 100\frac{1}{10000}^{300/149}\frac{300}{149}y^{151/149} = 1.60$$

Now the solution is $y = 750,000$ and $L(750,000) = 5,959$, for a total market employment of 29,795.

7. *Revolution_a.*

(a) The profit function is total revenue minus total cost. To find the total cost, we need to know how much steel is used per bumper, which is just the inverse of the production function: $S(q) = (0.001q)^{12}$. Then:

$$\Pi(q) = 500q - 800(0.001q)^{12}$$

(b) To maximize the profit function, take the derivative and set equal to 0:

$$\frac{d\Pi}{dq} = 500 - 12 \cdot 800(0.001q)^{11} \cdot 0.001 = 0$$
$$(0.001q)^{11} = \frac{500}{12 \cdot 800 \cdot 0.001} \Rightarrow q^* = 1432$$

In words, the condition is price (or marginal revenue) equals marginal cost.

(c) AG's method also starts with profits equal total revenue minus total cost, but they are all measured in terms of S. Since the amount of bumpers produced is always $q = 1000S^{\frac{1}{12}}$, AG's profit function is:

$$\Pi(S) = 500 \cdot 1000S^{\frac{1}{12}} - 800S$$

He also takes the derivative and sets equal to 0:

$$\frac{d\Pi}{dS} = \frac{1}{12}500 \cdot 1000S^{\frac{-11}{12}} - 800 = 0 \Rightarrow S^{\frac{-11}{12}} = \frac{800 \cdot 12}{500 \cdot 1000} \Rightarrow S = 74.6$$

In words, this condition is price of bumpers times the marginal product of steel equals the price of steel. If we evaluate the production function at AG's optimal S, we get $q = 1000(74.6)^{\frac{1}{12}} = 1432$, the same answer as in part (b).

(d) AG's method is identical. Because the production function provides a direct relationship between S and y, either decision making process yields the same result. McCoy's condition says that additional output should be produced until the cost of another unit equals the revenue from selling it. Barn's condition says that additional steel should be purchased until the cost of the steel equals the revenue generated from selling bumpers made from the steel. These conditions are restatements of the same idea.

8. *UncleKarl_a.*

(a) Assuming you get paid the coupon at the end of the year, the present value equation is:

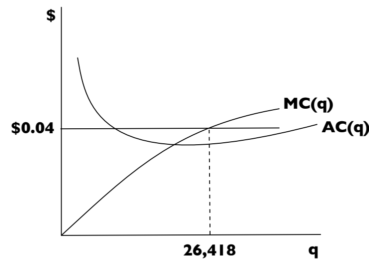
$$1000 = \frac{50}{1+i} + \frac{1050}{1+i} \Rightarrow 1000(1+i) = 1100 \Rightarrow i = 10\%$$

(b) We know that you can buy a risk-free bond and get a yield of 10%. Therefore any risk-free investment should have a cost of capital of \$0.10 per dollar invested. Presumably the online music business is very risky, so a cost of capital of \$0.15 would be more appropriate. (Indeed, a cost of capital of more like \$0.40 might be reasonable.)

(c) Since $q(K) = 10K^{9/10}$, you need $K(q) = \frac{q}{10}^{10/9}$ units of capital to produce output q. Since capital costs \$0.15, and you have an additional fixed cost of \$5000 that also comes out of capital, the cost curves are:

$$TC(q) = 0.15\left(5000 + \frac{q}{10}^{10/9}\right) = 750 + 0.0116q^{10/9}$$
$$AC(q) = \frac{TC(q)}{q} = \frac{750}{q} + 0.0116q^{1/9}$$
$$MC(q) = \frac{dTC(q)}{dq} = 0.0129q^{1/9}$$

If you draw the graph exactly, it is a little strange because marginal cost is concave:



(d) Your profit maximizing quantity is where marginal cost equals price:

$$MC(q) = p \Rightarrow 0.0129q^{1/9} = 0.04 \Rightarrow q^* = 26,418$$

At that quantity, you need to invest $K(26,418) = 6,340$ dollars of capital plus the 5,000 dollar startup cost. Given that your cost of capital is $0.15, your total costs are $1,701. Your total revenue is $pq^* = 0.04 \times 26,418 = \$1,056.72$. Thus you actually lose money on this investment, since your revenues are lower than your costs, including the proper cost of capital. You should buy the bond instead!