

ECON 110, Prof. Hogendorn

Problem Set 6 Answers

1. *LaborMarket_a*. Each firm has labor demand $\left(\frac{10p}{w}\right)^2$, so total labor demand is $L(w, p) = 1000 \left(\frac{10p}{w}\right)^2$. If $p = 10$, this simplifies to $L(w) = 10,000,000w^{-2}$. Total labor supply is 800,000, so labor market equilibrium is

$$10,000,000w^{-2} = 800,000 \Rightarrow w^{-2} = 0.08 \Rightarrow w = 3.54$$

2. *Generators_a*.

- (a) GenCo A's conditional factor demand for oil is found by inverting the production function:

$$540g = q^3 \Rightarrow g(q) = \frac{q^3}{540}$$

Profits are revenue minus fixed cost minus variable cost:

$$\Pi(q) = pq - TC(q) = pq - 20 - 200\frac{q^3}{540}$$

To maximize profits, take the derivative and set equal to 0. This is equivalent to the price-equals-marginal-cost condition.

$$p - MC(q) = p - 600\frac{q^2}{540} = 0$$

Finally, the supply curve is quantity as a function of price:

$$q^2 = \frac{9}{10}p \Rightarrow s(p) = \left(\frac{9}{10}p\right)^{1/2}$$

- (b) With this method, costs are easy, just $20 + 200g$. Revenue is equal to price times the amount of production:

$$\Pi(g) = pq(g) - 20 - 200g = p(540g)^{\frac{1}{3}} - 200g$$

To maximize profits, take the derivative and set equal to 0. This is equivalent to the price-times-marginal-product-equals-factor-price condition.

$$pMP_g - 200 = p \frac{1}{3} 540^{\frac{1}{3}} g^{-\frac{2}{3}} - 200 = 0$$

Now finding the supply curve takes two steps. First, find the unconditional factor demand $g(p)$, then use the production function to turn this into quantity produced as a function of price:

$$\begin{aligned} g^{-\frac{2}{3}} &= \frac{600}{p540^{\frac{1}{3}}} \\ g(p) &= \frac{600^{-3/2}}{p^{-3/2}540^{-1/2}} \\ s(p) = q(g(p)) &= 540^{1/3} \frac{600^{-1/2}}{p^{-1/2}540^{-1/6}} \\ s(p) &= 540^{1/2} \frac{600^{-1/2}}{p^{-1/2}540^{-1/6}} \\ s(p) &= 540^{1/2} p^{1/2} 600^{-1/2} \end{aligned}$$

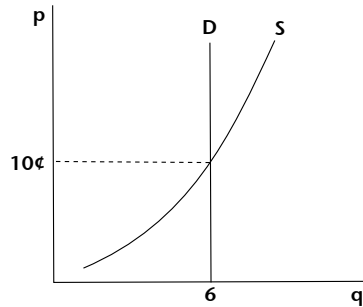
- (c) Both gencos make their decisions on the basis of “should the company do a little more.” GenCo A’s condition says that additional electricity should be produced until the marginal cost of another unit equals the revenue from selling it. GenCo B’s condition says that additional oil should be purchased until the cost of the oil equals the revenue generated from the marginal product (measured in electricity) made from the oil. These conditions are restatements of the same idea; both say “is it profitable to do a little more of this activity.”
- (d) Market equilibrium occurs when supply of electricity equals demand for electricity. Demand is just 6. Market supply is the sum

of the supply curves of GenCos A and B. Thus,

$$2s(p) = 2 \cdot 540^{(1/2)} p^{(1/2)} 600^{(-1/2)} = 6$$

$$0.95p^{(1/2)} = 3$$

$$p = 10¢$$



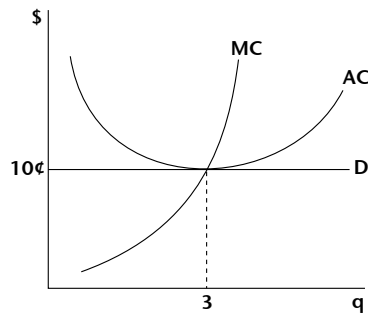
(e) We already found TC above, so applying the definitions of AC and MC gives:

$$TC(q) = 20 + 200 \frac{q^3}{540}$$

$$AC(q) = \frac{TC(q)}{q} = \frac{20}{q} + 200 \frac{q^2}{540}$$

$$MC(q) = \frac{dTC(q)}{dq} = \frac{600}{540} q^2$$

To draw the graph, we need to find that $AC(3) = 10$.

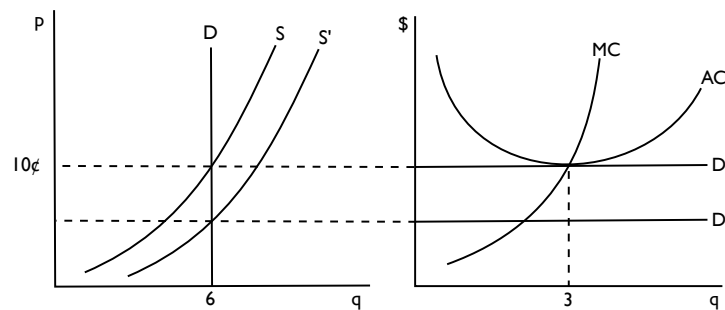


Since the average cost is equal to price, profit is:

$$\Pi(3) = (p - AC(3))3 = (10 - 10)3 = 0$$

Thus, there is no super-normal profit or loss.

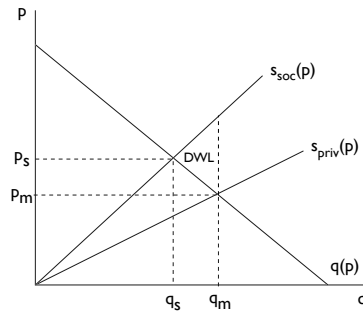
- (f) The market is already in long-run equilibrium! Since the gen-cos make zero profits, they have no reason to leave the industry (their capital would earn the same return elsewhere). And if a new firm entered, it would shift market supply to S' , lowering the price and changing the demand facing an individual firm to D' . Then price would be below average cost, and the firm would make losses. Thus, no firm would enter.



3. Boomerangs_a.

- (a) “I could choose the optimal quantity of boomerangs by setting marginal cost equal to price of boomerangs, or I could choose the optimal amount of wood by setting the price of wood equal to the price of boomerangs times the marginal product of wood.
- (b) The point is special, because it must be the lowest point on the average cost curve. At lower quantities, marginal cost is lower than average cost and pulls it down, while at higher quantities the marginal cost is higher than average cost and pulls it up.
- The firm doesn't particularly like this point, since there are no economic profits, but in a competitive market, it can expect that entry of new firms will push price down until price equals average cost. So in long-run perfectly competitive equilibrium, the firm will probably end up producing at this point.
- (c) Marginal social cost is higher than marginal private cost due to deforestation. The market quantity q_m is too large relative to the

social quantity q_s . The area marked DWL is the deadweight loss caused by the polluting over-production. If a Pigouvian Tax were enacted, it would shift the supply curve to the s_{soc} curve, resulting in a new price to consumers of p_s . The DWL would be avoided.



(d) Conditional factor demand is how much wood to make q boomerangs:

$$q = 5v^{1/3} \Rightarrow v^{1/3} = \frac{q}{5} \Rightarrow v(q) = \left(\frac{q}{5}\right)^3$$

To find the unconditional factor demand, we need to maximize profits first. The simplest way is the second method in part (a), set price of wood equal to the price of boomerangs times the marginal product of wood :

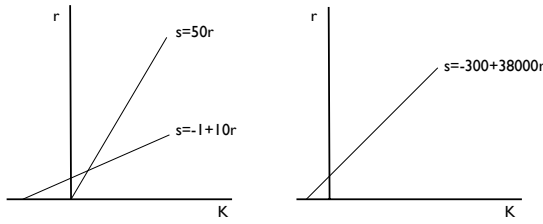
$$p_v = p \frac{dq(v)}{dv} \Rightarrow p_v = p \frac{5}{3} v^{-2/3} \Rightarrow v^{-2/3} = \frac{3p_v}{5p} \Rightarrow v(p_v) = \left(\frac{5p}{3p_v}\right)^{3/2}$$

4. *SmallCountry_a.*

(a) Aggregate saving is

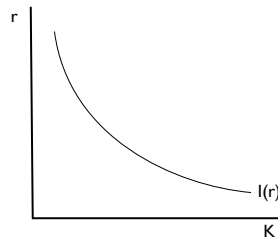
$$700 \cdot 50r + 300(-1 + 10r) = -300 + 38,000 \cdot r$$

When the interest rate is 3%, the savings of the two types and the aggregate are: 1.5, -0.7, 840. When the interest rate is 11%, these change to: 5.5, 0.1, 3880.



At the low interest rate, 30% of households borrow and 70% save. There is more than enough saving to fund the borrowing, so aggregate saving is positive. When the interest rate rises, the savers save more, and the borrowers find it more expensive to borrow and actually switch over to saving. The aggregate saving obviously rises.

(b) $I(r) = 100i(r) = 1,000/r$.



(c) Setting aggregate saving equal to aggregate investment gives:

$$-300 + 38,000 \cdot r = 1,000/r \Rightarrow 38,000 \cdot r^2 - 300r - 1,000 = 0$$

The root of this is $r = 16.6\%$.

(d) Every country has a menu of investment projects that could be undertaken. The projects with payoffs above the prevailing interest rates will be implemented; the rest will not. So all other things equal, a country with a higher interest rate will have less investment, less capital deepening, and therefore lower growth.