

ECON 110, Prof. Hogendorn

Problem Set 6

1. *OldGermans*. In Germany, the birth rate is low and the population is ageing. As a result, the working age population is falling at about 0.2% per year. It has been suggested that this population decline puts the German economy at risk. This question asks you to use our simple neoclassical model to evaluate that claim.

Let there be $\mathcal{L} = 243$ German workers who inelastically supply labor and who spend all of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. (Aggregate meaning we treat all the firms as if there were just 1.) Let $p = 1$.

- (a) Find the equilibrium real wage in the labor market and graph the labor market.
 - (b) Verify that there is also equilibrium in the beer market and graph the production function. What share of workers' income comes from wages and what share from dividends?
 - (c) Suppose that over 10 years, the German population falls and there are only $\mathcal{L}' = 198$ workers. Find the new general equilibrium.
2. *Uchitelle*. The following are quotes from an op-ed by Louis Uchitelle that appeared in the New York Times on August 25, 2002. It provides some food for thought, but we can evaluate the arguments a lot more clearly in a simple macroeconomic model. At the end of the article is a suggestion for trying to model Uchitelle's idea.

In Alice in Wonderland fashion, we talk of expansion and ignore the contraction all around us. We convince ourselves that out of cost-cutting will come prosperity. But while cost-cutting can lift a single company or two, when practiced widely enough it can pull down an economy. And that is happening today.

... consider what happens in an imaginary country where Burger King and McDonald's are the entire business sector and the total national output 100 hamburgers a day, evenly divided between the companies matches the demand from this nation's consumers. Demand and sales revenue, however, stay flat. So Burger King lays off two workers and uses the saved wages partly to fatten profits and partly to discount prices by just enough to take sales and revenue away from McDonald's. And McDonald's responds in kind. But soon, the four laid-off workers, with little income, buy fewer hamburgers, and the nation's total consumption drops to 95 hamburgers a day. That sets off another round of cost-cutting and price discounting, and our imaginary nation sinks gradually into stagnation or deep recession not unlike America in the 1930's.

Let the entire population of the economy be 32 workers who inelastically supply labor and who spend all of their income on hamburgers. Let McDonald's and Burger King be identical firms that each have production function $f(L) = 25L^{0.25}$. Let them both behave as perfect competitors. Let $p = 1$.

- (a) Find the equilibrium real wage in the labor market. Remember that there are TWO firms, so the total demand for labor is the sum of each firm's demand for labor. Illustrate with a graph.

- (b) Verify that there is also equilibrium in the hamburger market and comment on the sources of the workers' total income.
- (c) Suppose that the two firms each laid off 2 workers as Uchitelle wrote. Assume the laid-off workers get no income whatsoever. Also suppose that the remaining 28 workers receive the same wage as before. Show the situation on a labor market diagram. Are the firms' profits higher? What about the workers' incomes?
3. *Fear-goods*. This problem shows how in the neoclassical long-run macro model, widespread fear across an economy will not cause a recession! This is an important and comforting insight for the long run, but on the other hand, in the long run we are all dead... Suppose the production function for the one representative firm in the economy is $Y = f(L) = 20L^{4/5}$. There are $\mathcal{L} = 40$ workers who inelastically supply labor.
- (a) Show that the labor demand curve is $L(w) = (16/w)^5$, graph the labor market, and show the equilibrium real wage.
- (b) Verify the national income accounts identity, i.e. that income from wages and dividends (which equals consumption) equals output (all of which is also consumption).
- (c) Now suppose that people in this country hear about the financial crisis. Everyone becomes very fearful of the future. The firm shifts down its labor demand curve to $L(w) = (8/w)^5$ – even though this is not profit maximizing because the production function remains unchanged. Assuming the labor market still clears, what happens to the wage, income from wages, income from dividends, and output?
4. *GrowingChina*. This problem discusses the Malthusian trap that has worried China for centuries and that the country now seems

to have escaped. Let there be $\mathcal{L} = 1000$ Chinese workers who inelastically supply labor and who spend all of their income on rice. These workers own the Chinese rice firms which have aggregate production function $Y = f(L, K) = A(hL)^{2/3} K^{1/3}$. (Aggregate meaning we treat all the firms as if there were just 1.) Let $A = 3.33$, $h = 1$, $p = 1$ and let $K = 729$. Note that the Chinese capital stock is constant until part (d) of this problem.

- (a) Find the equilibrium real wage and graph the labor market.
- (b) Verify that there is also equilibrium in the rice market and graph the production function. What is output per worker (Y/\mathcal{L})?
- (c) Suppose that over several years, the Chinese workforce rises to 1,728 workers. If nothing else changes, what is the new general equilibrium (the new wage and the new output per worker)? Why don't these new workers produce enough to keep the output per worker at least as high as before?
- (d) Consider the following changes to the production function: an increase in A , an increase in K , and an increase h . How would each of these help China escape the Malthusian trap? What is the name for each of these sources of growth?

Review Problems only, not to turn in:

5. *Deflate*. Given the information below about the U.S. economy, how much did real GDP grow between 1980 and 1990? Between 1990 and 2000?

	1980	1985	1990	1995	2000
Nominal GDP (trillions)	2.8	4.21	5.8	7.4	9.96
GDP deflator (1996=100)	57.0	73.7	86.5	98.1	106.9

6. *OldGermansSave*. As in *OldGermans*, there are 243 German workers who inelastically supply labor, but now they save 100 beers (in total) for the future and spend the rest of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. The German beer firms have aggregate investment demand of $I = 1200/r$, where r is the real interest rate. Let $p = 1$.
- Find the equilibrium real wage in the labor market and graph the labor market. Verify that there is also equilibrium in the beer market and graph the production function.
 - Graph the capital market. What is the equilibrium real interest rate?
 - What happens if Germans become more pessimistic and start saving 110 beers?
 - Go back to just 100 beers saved. What happens if the German government levies taxes of 60 beers but German Chancellor Angela Merkel drinks 80 beers?
7. *Botswana*. Let's consider the IMF's policy toward Botswana (criticized by Joseph Stiglitz in his book *Globalization and its Discontents*, pg. 38). Normalize Botswana's working population to $\mathcal{L} = 100$. Let Botswana initially have a production function $y = f(L) = 380L^{1/2}$, and assume the firms represented by this function are owned by the workers. Set the price of y equal to 1.
- Find the equilibrium real wage in Botswana's labor market and graph the labor market. Also graph the production function.
 - Private Botswanans were saving 700. The government was spending 25% of GDP and collecting taxes of 25% of GDP. Firms' investment demand function was $I = \frac{7630}{r}$. We will

ignore foreign capital flows (actually, they were considerable in real life). Graph the capital market, showing the private, government, and total savings curves and the investment curve. What was the real interest rate?

- (c) Stiglitz claims that Botswana faced two negative shocks in 1981 due to drought and problems in the diamond industry. We'll model this by saying that the production function changed for the worse to $y = f(L) = 350L^{1/2}$. What was the new real wage and the new output?
- (d) The government of Botswana collected taxes of 25% of the new, lower real GDP, but it did not take the IMF's advice and continued to spend the same real amount as before the shocks. Real private saving and investment demand remained unchanged. Graph what happened in the capital market and find the new real interest rate.
- (e) The neoclassical model suggests that real wages in Botswana will grow more slowly as a result of the government's decision in (c). Why? What is an argument against this view?

Answers to Review Problems:

5. *Deflate_a*. Total growth in real GDP between 1980 and 1990 was 36.5% and between 1990 and 2000 was 38.9%.

6. *OldGermansSave_a*.

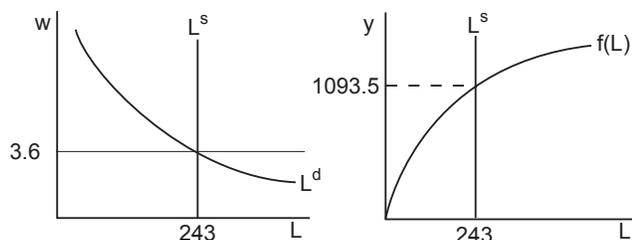
(a) We can find labor demand using $pMP_L = w$, so,

$$1 \cdot \frac{4 \cdot 54}{5 \cdot 4} L^{-1/5} = w \Rightarrow L^d = \left(\frac{54}{5w} \right)^5$$

Setting $L^d = L^s = 243$ gives an equilibrium real wage of $w = 3.6$.

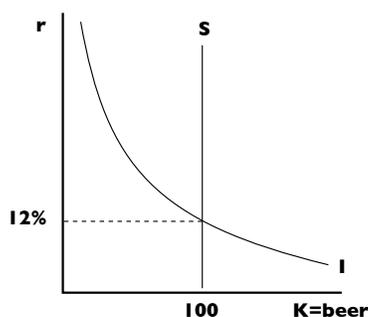
The total costs of the firm are $wL = 3.6 \cdot 243 = 874.8$. The total revenues are $py = 1 \cdot f(243) = 1093.5$. Thus the profits, paid as dividends, are 218.7. The firm's output is 1093.5.

Workers earn total wages of $wL = 874.8$ and total dividends of 218.7. Their total consumption of beer is thus $1093.5 - 100 = 993.5$, and the remaining 100 beers are saved, so there is equilibrium.



(b) The equilibrium real interest rate is found by setting

$$I = S \Rightarrow \frac{1200}{r} = 100 \Rightarrow r = 12\%$$



(c) The higher savings reduces the real interest rate:

$$I = S \Rightarrow \frac{1200}{r} = 110 \Rightarrow r = 10.9\%$$

(d) Consumption of beers falls to $1093.5 - 100 - 60 = 933.5$. Private savings stays the same at 100. Government spending is 80, so $T - G = -20$, i.e. the government runs a deficit. National saving is then $100 - 20 = 80$. The real interest rate rises to

$$I = S \Rightarrow \frac{1200}{r} = 80 \Rightarrow r = 15\%$$

Note that the government deficit fully crowds out private investment, which falls from 100 to 80.

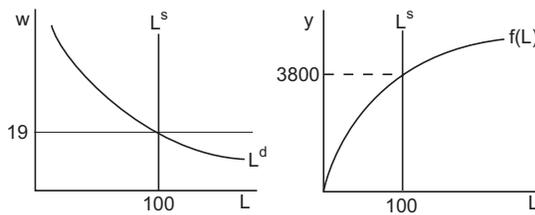
7. Botswana_a.

- (a) Labor demand can be found quickly by remembering that $pMP_L = w$, so

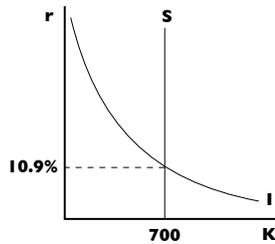
$$190L^{-1/2} = w \Rightarrow L^{1/2} = \frac{190}{w} \Rightarrow L^D(w) = \frac{36100}{w^2}$$

Then setting labor demand equal to labor supply gives us:

$$L^D(w) = L^S \Rightarrow \frac{36100}{w^2} = 100 \Rightarrow w = 19$$



- (b) Government spending and taxes are the same, so government saving is zero. Thus, capital market equilibrium occurs where private saving, S^P equals I : $\frac{7630}{r} = 700 \Rightarrow r = 10.9\%$



- (c) We have to find the new labor curve:

$$175L^{-1/2} = w \Rightarrow L^{1/2} = \frac{175}{w} \Rightarrow L^D(w) = \frac{30625}{w^2}$$

Then setting labor demand equal to labor supply gives us:

$$L^D(w) = L^S \Rightarrow \frac{30625}{w^2} = 100 \Rightarrow w = 17.5$$

So real wages fall to 17.5 and output falls to $f(100) = 3500$.

- (d) The government is still spending $25\% \cdot 3800 = 950$ but is only collecting taxes of $25\% \cdot 3500 = 875$. Thus it is running a deficit of 75, which raises the equilibrium real interest rate to 12.2%.

(On a graph, there should be a vertical line for government saving at -75 and a vertical line for total saving at 625. The investment demand is the same.)

- (e) The neoclassical model says that the lower private investment will lead to less capital deepening and therefore slower economic growth. A counter-argument is that in the midst of its problems, government spending is needed to stabilize education, healthcare, and so forth, and that these provide more improvements to the production function than private capital.