ECON 110, Prof. Hogendorn

Review Problem Set 9

1. Water. The government has offered to give you a monopoly if you will provide water to a city. The inverse demand curve is $p(q) = 1000 - 0.01q$ and the average cost curve is $AC(q) = \frac{25{,}000{,}000}{q} + 100$.

    (a) What are the marginal revenue and marginal cost curves?

    (b) What is the optimal price you should charge and quantity you should produce? What is the profit of the monopolist?

    (c) Graph this situation carefully.

    (d) If the government were to give this firm a lump-sum subsidy, how big should it be if (1) the government is concerned only about its own budget or (2) it is concerned with overall welfare?

2. ChinaMobile. This problem is loosely based on reality: Every year, cellular phone equipment becomes cheaper, and China Mobile's costs fall. Specifically, assume that in year 1, the marginal cost per minute is 0.20 yuan and in year 2 it falls to 0.10 yuan. (Note, in both years, MC is constant, i.e. horizontal.)

    (a) Let demand (in minutes per typical consumer) be given by $y(p) = 100 - 100p$. Treating China Mobile as a monopoly, what is the profit maximizing price and number of minutes in year 1? What about year 2? On the same graph, show the optima in both years.

    (b) Suppose that China Mobile committed to the quantities from (a) in year 1 and year 2, but that the demand estimate turned out to be a mistake. Really demand is $y'(p) = 60 - 60p$. In terms of foregone profits, are China Mobile's problems getting worse or better over time?

(c) From the point of view of China as a whole, was the mistake bad or good? In money terms, how much did China gain or lose in year 2? Illustrate on a graph.

(d) Just for fun: Who do you think China Mobile hired to do the initial demand estimate?

Answers:

1. Water_a.

(a) Total revenue and marginal revenue are:

$$\begin{aligned} TR(q) &= (1000 - 0.01q)q \\ MR(q) &= 1000 - 0.01q - 0.01q = 1000 - 0.02q \end{aligned}$$

Total cost and marginal cost are:

$$\begin{aligned} TC(q) &= 25,000,000 + 100q \\ MC(q) &= 100 \end{aligned}$$

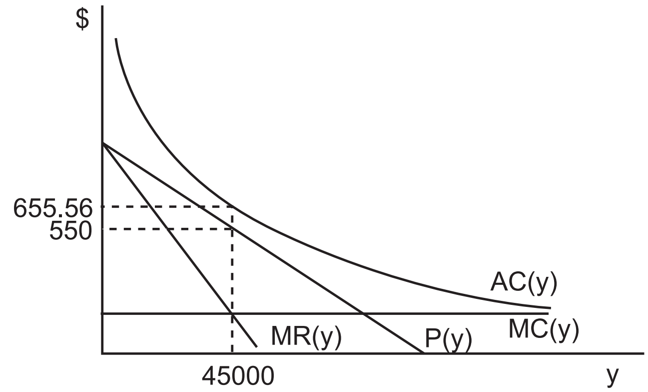
(b) The optimal behavior is to set marginal revenue equal to marginal cost:

$$\begin{aligned} 1000 - 0.02q &= 100 \\ q &= 45000 \end{aligned}$$

The price at this output is $p(45,000) = 1000 - 0.01 \cdot 45,000 = 550.$ The profit is:

$$\begin{aligned} \Pi(45,000) &= (p(45,000) - AC(45,000))45,000 \\ &= (550 - 655.56)45,000 \\ &= -4,750,000 \end{aligned}$$

(c) The key here is that the AC curve lies everywhere above the demand curve, so there's no way the monopoly can avoid a loss, even at the maximum "profit" level.



(d) If the government is only concerned with its own budget, the cheapest lump-sum subsidy needed is $4,750,000, which is just enough to offset the monopoly lost. We know this is the smallest possible lump-sum subsidy that will induce you to provide water service, because the monopoly output maximizes profits, or, in this case, minimizes losses.

If the government is concerned with overall welfare, it should induce the monopoly to set price equal to marginal cost. At this price, the monopolist makes $p(q) = 100 \Rightarrow q = 90,000$ units and its profits are:

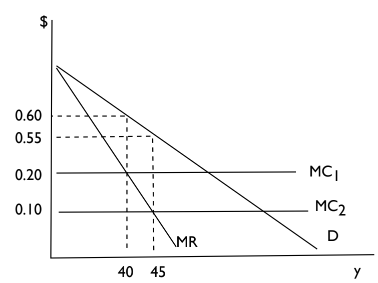
$$
\begin{aligned}
\Pi(90,000) &= (p(90,000) - AC(90,000))90,000 \\
&= (100 - 377.78)90,000 \\
&= -25,000,000
\end{aligned}
$$

Thus the government would have to provide a $25,000,000 subsidy on condition that the firm produces 90,000 units. This would be harder on the government budget, but it would maximize social welfare.

2. ChinaMobile_a.

(a) The inverse demand curve is $p(y) = 1 - 0.01y$. For a monopoly, profit is maximized when marginal revenue equals marginal cost. $TR = p(y)y = y - 0.01y^2$, so marginal revenue is $MR = 1 - 0.02y$. Then in year 1 the profit maximizing quantity is $1 - 0.02y = 0.2 \Rightarrow y = 40$. The price at this quantity is $p(40) = 0.60$.

In year 2, the same calculations with the new marginal cost give $1 - 0.02y = 0.1 \Rightarrow y = 45$ and $p(45) = 0.55$. The graph looks like this:



(b) The true inverse demand curve turns out to be $p'(y) = 1 - 0.017y$. In year 1, they mistakenly set $y = 40$. This gives them a price of $p'(40) = 0.32$. Their profit is $\pi = py - TC(y) = 0.32 \times 40 - 0.2 \times 40 = 4.8$.

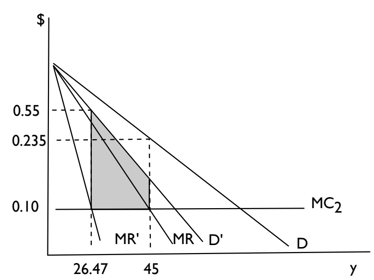
Actually, marginal revenue was $1 - 0.034y$, so the correct monopoly quantity was $1 - 0.034y = 0.2 \Rightarrow y' = 23.5 \Rightarrow p'(23.5) = 0.6$. The profit would have been $\pi' = 0.6 \times 23.5 - 0.2 \times 23.5 = 9.4$. Thus, China Mobile forewent $\pi' - \pi = 9.4 - 4.8 = 4.6$ profit.

In year 2, they mistakenly set $y = 45$. The price is $p'(45) = 0.235$. Their profit is $\pi = py - TC(y) = 0.235 \times 45 - 0.1 \times 45 = 6.075$.

Actually, the correct monopoly quantity was $1 - 0.034y = 0.1 \Rightarrow y' = 26.47 \Rightarrow p'(26.47) = 0.55$. The profit would have been $\pi' = 0.55 \times 26.47 - 0.1 \times 26.47 = 11.91$. Thus, China Mobile forewent $\pi' - \pi = 11.91 - 6.075 = 5.835$ profit.

Thus, not only did they lose a lot in both years (about half of potential profits), but things were worse in year 2 than in year 1. The reason is that there is more to lose when the monopoly has lower costs it can take advantage of.

(c) Since monopolies inefficiently reduce quantities below the competitive level, and since price was still above marginal cost despite the mistake, we can be sure that China as a whole gained from the mistake. In year 2, 45 units were produced instead of 26.47. The added value (reduced deadweight loss) of these units was the area between the demand curve and the marginal cost curve between 26.47 and 45 units, shaded on the graph below.



The numerical gain was:

$$\begin{aligned}\int_{26.47}^{45} 1 - 0.017y - 0.2dy &= \left|0.8y - .0085y^2\right|_{26.47}^{45} \\ &= 18.7875 - 15.22 \\ &= 3.5675\end{aligned}$$

(d) Amherst Guy!