ECON 110, Prof. Hogendorn

Problem Set 2

1. *SUVs.* This question asks you to analyze the market for Sport Utility Vehicles (SUVs) using a *nonlinear* demand curve.

   (a) The demand function (measured in hundreds of thousands of vehicles) for SUVs turns out to be $q^d = 4027p^{-1.5}$, where p is the price of a typical SUV (in this problem we will measure price in tens of thousands of dollars). What are the first and second derivatives of this function? Graph the function and explain how the first and second derivatives relate to the shape of the graph.

   (b) The supply of SUVs turns out to be $q^s = 258.3p$. What is the equilibrium price and quantity?

   (c) Suppose that the price of gas rises. Which of the following is more likely to be the new demand curve for SUVs? Why?

   $$q^d = 4300p^{-1.5} \qquad q^d = 3700p^{-1.5}$$

   (d) Calculate and graph what happens to the equilibrium price and quantity after the demand curve changes.

2. *Shifters.* Illustrate and explain the impact on equilibrium market price and quantity exchanged of each of the following changes:

   (a) An improvement in the technology of production

   (b) An increase in individuals' desire for the good

   (c) A decrease in the wage paid to all workers (be careful here)

3. *MTA.* On December 30, 2010, the fare for one subway ride in New York City was raised from $2.25 to $2.50. Annual ridership is about 1.6 (measured in billions).

Suppose that demand turns out to be

$$Q(p) = 2.21p^{-0.4}$$

(a) Graph this demand function and show the price/quantity point where the price of a ride is $2.25.

(b) Find the elasticity of demand using the derivative.

(c) Will the increase in fare to $2.50 increase or decrease revenue in the short run? Can you justify your answer without actually finding the new revenue?

(d) Suppose that you find out that ridership in 2011 is *higher* than 1.6. What do you think is the most likely explanation for such a finding? Illustrate your answer with your graph.

4. *Juvenor.* You take a job at a pharmaceutical market research firm. On your first day, the woman in the cubicle next to you says, "You'd better watch yourself – there was some guy from Amherst here before you, and he only lasted a week." On your desk you find some handwritten notes:
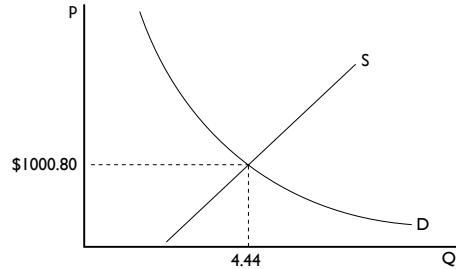
Assignment: find market equilibrium for Juvenor (drug that makes people feel younger) and find consumer surplus.

Data: demand from men: $p = 100 - .02q$, demand from women: $p = 4000q^{-1}$, supply: perfectly inelastic, $q = 1000$.

Solution:

(a) Find market demand: men+women $= 100 - .02q + 4000q^{-1}$.

(b) set equal to supply: $1000 = 100 - 0.02q + 4000q^{-1} \Rightarrow q = 4.44$

(c) Draw graph:

(d) Find consumer surplus: $CS = \int_0^{4.44} 100 - 0.02q + 4000q^{-1} dq$

At this point, there is some scratch work on the integral, and then a line trailing off to the lower right corner of the page. Your boss tells you that the data are correct, but you should redo each step of the solution, and explain what mistakes your predecessor made.

Review Problems only, not to turn in:

5. *psquared.* Suppose the demand function for a good is $q = 100 - 2p^2$.

   (a) Find the first and second derivatives of this demand function. What are the signs of the derivatives?

   (b) Graph this demand function. Explain how your answer to part (a) affects the shape of the curve.

6. *Healthcare.* Recently in my e-mail, I received notice of a new article published in an economics journal. The article is entitled "A theoretical rationale for an inelastic demand for health care."

   (a) OK, you've only taken a few days' worth of economics, but can you provide a theoretical rationale for inelastic demand for health care?

   (b) Draw a supply and demand diagram for health care making both functions linear. Draw the diagram so that at the equilibrium, demand is inelastic and supply is perfectly elastic.

3

(c) If costs rose in the health care industry, show what would happen in your diagram, and discuss the relative size of the changes in the quantity and price of health care.

7. *Textbooks.* Suppose the market supply curve for economics textbooks is given by $s(p) = 10p$ and the market demand by $q(p) = 100 - 10p$. Derive and graph the competitive equilibrium price, quantity exchanged, and consumer and producer surplus.

8. *Five-Households.* Suppose there are 5 households, each with demand curve $q(p) = 10p^{-2}$. Derive and graph the market demand curve. What is the total consumer surplus when $p = 2$?

9. *London.* In February 2003, drivers entering central London began paying a toll of £5 to help mitigate congestion. Previously there was no toll, and about 250,000 cars entered central London each day. The toll cut traffic by 15% and the city uses the toll revenue to fund public transportation projects.

(a) Using the data above, find an approximate linear demand curve for automobile access to central London. (Hint: do not try to use elasticity to do this.)

(b) Using your demand curve, what is the approximate price elasticity of demand with the £5 toll in place?

(c) What was the price elasticity of demand without the toll? Do not use any math to answer this question.

(d) How much revenue does the toll generate? Do you know whether London could obtain more revenue by raising the toll?
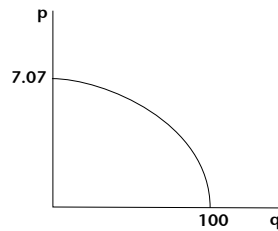
Answers to Review Problems:

5. *psquared_a.*

(a) The derivatives are:

$$\frac{dq}{dp} = -4p < 0 \qquad \frac{d^2q}{dp^2} = -4 < 0$$

(b) From (a) we know that the function is downward sloping and concave:

6. *Healthcare_a.*

(a) Healthcare has very few substitutes because sick people have very limited alternatives. The only source of any elasticity is people delaying elective and preventive care and the poorest people going without care altogether.

(b) Supply must be horizontal and it must intersect the demand curve below its midpoint.
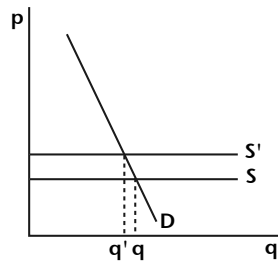
(c) Higher costs in the industry shift the supply curve up. At the new equilibrium, the percentage increase in price is greater than the percentage increase in supply due to the fact that demand is inelastic.

7. *Axolotls_a.*

(a) The graph is:
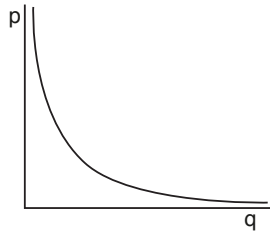
(b)

$$\varepsilon = \left| \frac{dq}{dp} \frac{p}{q} \right| = \frac{A}{p^2} \frac{p}{\frac{A}{p}} = 1$$

This is an example of a *constant elasticity demand function* along which elasticity does not vary with price. In this case, it's unit elastic along the entire curve.

(c) On this curve, total expenditure is $pq = p\frac{A}{p} = A$. Thus total expenditure on the good is A regardless of the price. This makes sense since the curve is unit elastic: total spending/revenue does not change as price changes.

(d) Suppose z increases. We find the effect on demand by taking the derivative with respect to z :

$$\frac{dq}{dz} = -\frac{A}{(p+z)^2} < 0$$

Since the derivative is negative, an increase in z reduces demand for axolotls which means the goods are complements.

8. *Textbooks_a.* Supply equals demand when $10p = 100 - 10p$, or $p = 5$. At this price, $q = 50$. The choke price is 10; thus consumer surplus is $\frac{1}{2}(10 - 5)50 = 125$. Producer surplus is $\frac{1}{2}(5 - 0)50 = 125$ as well.
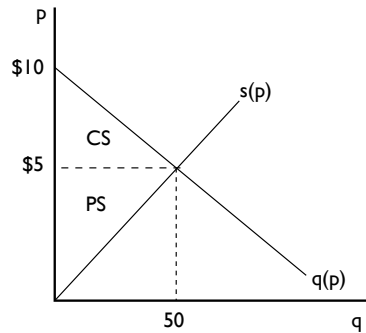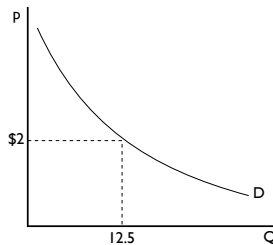
9. *Five-Households_a.* We can simply add quantities up (horizontal addition in the graph). Thus the market demand function is 5 times the individual demand function, or $q(p) = 50p^{-2}$.

To find the consumer surplus, note there is no choke price, and therefore the integral is improper. But the answer is:

$$\int_2^\infty 50p^{-2} = \lim_{t\to\infty} \int_2^t 50p^{-2} = \lim_{t\to\infty} -50t^{-1} + 50 \cdot 2^{-1} = 0 + 25 = 25$$

The graph is:

10. *London_a.*

(a) The original point was $(p, q) = (0, 250000)$. Finding that 15% of 250,000 is 37,500, the new point is $(5, 212500)$. The equation for a linear demand curve that connects these points is:

$$q(p) = 250,000 - 7,500p$$

(b) $\epsilon = \left| \dfrac{dq}{dp} \dfrac{p}{q} \right| = 7,500 \dfrac{5}{212,500} = 0.16$

(c) Demand is linear, and without the toll we are at the bottom of a linear demand curve where elasticity is 0. Alternatively, if price is 0, any increase in price is an infinity percent increase, and any percent change in quantity divided by infinity is 0.

(d) The toll generates $5 \cdot 212,500 = £1,062,050$ in revenue. Since demand is inelastic at this point, increasing the toll will increase revenue.