ECON 110, Prof. Hogendorn

Problem Set 7

1. *Campbell.* An article in *Business Week,* Nov. 24, 2010, describes how Campbell Soup works very hard to improve productivity at its factories.

   (a) The article says that "Productivity gains come when hundreds of nitpicky efforts combine to save time, money, and effort." Draw how these efforts change the production function $f(L)$.

   (b) Suppose that Campbell is able to change its production function from $f(L) = 75L^{\frac{1}{2}}$ to $f(L) = 85L^{\frac{1}{2}}$. What is its labor demand curve for both cases?

   (c) The article focuses on the Campbell factory in the town of Maxton, NC. Draw the labor market of Maxton. Justify the way you draw the labor supply curve and the labor demand curve for the town. (Note Campbell is not the only employer in Maxton.)

   (d) The article says that "The efficiency gains make it less likely that recession-casualty jobs will come back." Later in the article, it also mentions that "In the latest quarter, [Campbell's] earnings fell 8.2 percent because of declining soup sales." With reference to your labor market diagram, which statement more likely explains unemployment in Maxton?

   (e) Suppose that Campbell has a near monopoly on the American soup market, with a demand curve of $p(q) = 5 - 0.011q$ Also suppose that Campbell must pay a wage of 10 and has the production function $f(L) = 85L^{\frac{1}{2}}$ mentioned above. What is the monopoly profit maximizing quantity and price?

(f) Draw a diagram of the monopoly profit maximization problem. Add an average cost curve that shows Campbell making a *loss* at the monopoly profit maximum. How could this be? Should Campbell Soup shut down if this happens?

2. *Revolution.* After Wesleyan, you take a job with McCoy consulting. It was a tough decision because McCoy's big rival, Barn & Co., was also recruiting you. And now the pressure is on because you are making a big presentation to Dolty, an auto parts manufacturer which is a perfectly competitive firm.

(a) The perfectly competitive price of a car bumper is $500. Dolty uses steel to make bumpers according to the production function $f(S) = 1000S^{\frac{1}{12}}$ where S is tons of steel. The price of steel is $800 per ton. What is Dolty's profit function $\Pi(q)$?

(b) Describe the condition for profit maximization that shows how many bumpers Dolty should produce.

(c) After you have shown the above, a team from Barn & Co. bursts into the room. Their young leader, known only by his initials A.G., says "Barn has a revolutionary new way to manage your firm. Don't think about bumpers, like these dinosaurs from McCoy! Instead, decide how much steel to buy!" He proceeds to write down a profit function $\Pi(S)$. Assuming he does this correctly, what does he write down? Show the condition for profit maximization using this function.

(d) Now it's up to you to save McCoy's reputation. Argue (in words) that the profit maximization condition for A.G.'s method is exactly the same as the profit maximization condition in your method, and that Barn & Co. has no revolutionary management technique.

3. *SmallCountry.* Remember that a country's supply of loanable funds is the *net* supply after households that borrow are subtracted from those who save. Suppose there is a small country with 1000 households. 700 of these have a savings function $s = 50r$, where r is the rate of return on capital. The remaining 300 households have savings function $s = -1 + 10r$. (You can imagine that both the number of households and the amount of savings are in thousands.)

   (a) Graph the individual and aggregate savings functions. Describe in words what happens to both types of household and the whole country when the interest rate rises from 3% to 11%.

   (b) There are 100 firms, and each firm has an investment demand function $(i(r) = 10/r$. Find and graph the aggregate investment function for the whole country.

   (c) Show that the equilibrium interest rate in this country is 16.6% (rounded to one decimal).

   (d) In most countries, a real interest rate of 7% would be more typical. Do you think this country will have higher or lower economic growth than the typical country? Explain.

4. *OldGermans.* In Germany, the birth rate is low and the population is ageing. As a result, the working age population is falling at about 0.2% per year. It has been suggested that this population decline puts the German economy at risk. This question asks you to use our simple neoclassical model to evaluate that claim.

   Let there be $\mathcal{L} = 243$ German workers who inelastically supply labor and who spend all of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. (Aggregate meaning we treat all the firms as if there were just 1.) There is no money, so the price of one beer is one beer.

(a) Find the equilibrium real wage in the labor market and graph the labor market.

(b) Verify that there is also equilibrium in the beer market and graph the production function. What share of workers' income comes from wages and what share from dividends?

(c) Suppose that over 10 years, the German population falls and there are only $\mathcal{L}' = 198$ workers. Find the new general equilibrium.

Review Problems only, not to turn in:

5. *MBAs.* The last recession was very hard on the strategic consulting industry. Firms like McKinsey, Bain, and Booz Allen & Hamilton laid off 30% of their workforce.

There were two components to the downturn. First, demand fell dramatically, in large part because of the demise of the dot-coms. Second, more executives began to have business school degrees and/or experience with the consulting firms. This made the ``sage advice" of the consultants themselves less useful and effectively reduced the marginal product of laborers with MBA (Master of Business Administration) degrees (see *The Economist*, 11/2/02, pg. 61).

For this problem, assume that the wage of MBAs is $100. (Note: for more realism, you can think of all money amounts in this problem in thousands.)

(a) Let a typical consulting firm have production function $f(L) = 10000L^{1/2}$ and the firm also incurs a fixed cost of 1000. What is this firm's total cost function, average cost function, average variable cost function, and marginal cost function?

(b) Graph these curves.

(c) If the price of consulting is $p = 2$ and there are 5 consulting firms, how many MBAs are hired?

(d) Suppose that p falls to 1.60 and also the production function changes to $f(L) = 10000L^{149/300}$. Now how many MBAs are hired?

6. *UncleKarl*. Your Uncle Karl gives you 20,000 dollars of capital.

(a) For $1000, you can buy a risk-free government bond with a coupon of $50 (payable at the end of the year), a face value of $1050, and a maturity of one year. What is the yield on this bond?

(b) Alternatively, you can invest some of the capital in a business venture providing downloadable music. For each dollar of capital invested over the course of one year, do you think it is more reasonable to let your cost of that capital be $0.05, $0.10, or $0.15? Discuss your answer with reference to part (a), assuming you can buy fractional amounts of the bonds.

(c) To simplify, assume no labor is involved in this business; the only factor is capital. Your production function is $f(K) = 100K^{9/10}$, where output is measured in the number of downloads. You must also use $5,000 more of capital to pay a fixed cost to get started. What are the equations for your total, average, and marginal cost curves, using your answer to (b)? Graph the AC and MC curves.

(d) If each download brings you revenue of $0.04, how much capital should you invest in this business? Show this on your graph. Do you earn a competitive rate of return on your capital, or do you receive rents?

7. *Lula*. Suppose there is a Brazilian government bond with a face value of R$100 (i.e. 100 reals, the currency of Brazil). The bond has a coupon of R$5 and matures in 1 year.

(a) If the bond's current price is R$80, what is its yield?

(b) Many investors thought that if Lula da Silva were elected president of Brazil, Brazilian debt would become more risky. Explain what probably happened to the price of Brazilian government bonds when Lula won.

8. *Deflate.* Given the information below about the U.S. economy, how much did real GDP grow between 1980 and 1990? Between 1990 and 2000?

	1980	1985	1990	1995	2000
Nominal GDP (trillions)	2.8	4.21	5.8	7.4	9.96
GDP deflator (1996=100)	57.0	73.7	86.5	98.1	106.9

9. *Uchitelle.* The following are quotes from an op-ed by Louis Uchitelle that appeared in the New York Times on August 25, 2002. It provides some food for thought, but we can evaluate the arguments a lot more clearly in a simple macroeconomic model. At the end of the article is a suggestion for trying to model Uchitelle's idea.

> In Alice in Wonderland fashion, we talk of expansion and ignore the contraction all around us. We convince ourselves that out of cost-cutting will come prosperity. But while cost-cutting can lift a single company or two, when practiced widely enough it can pull down an economy. And that is happening today.
>
> … consider what happens in an imaginary country where Burger King and McDonald's are the entire business sector and the total national output 100 hamburgers a day, evenly divided between the companies matches the demand from this nation's consumers. Demand and sales revenue, however, stay flat. So Burger

King lays off two workers and uses the saved wages partly to fatten profits and partly to discount prices by just enough to take sales and revenue away from McDonald's. And McDonald's responds in kind. But soon, the four laid-off workers, with little income, buy fewer hamburgers, and the nation's total consumption drops to 95 hamburgers a day. That sets off another round of cost-cutting and price discounting, and our imaginary nation sinks gradually into stagnation or deep recession not unlike America in the 1930's.

Let the entire population of the economy be 32 workers who inelastically supply labor and who spend all of their income on hamburgers. Let McDonald's and Burger King be identical firms that each have production function $f(L) = 25L^{0.25}$. Let them both behave as perfect competitors. Let the price of one burger be one burger.

(a) Find the equilibrium real wage in the labor market. Remember that there are TWO firms, so the total demand for labor is the sum of each firm's demand for labor. Illustrate with a graph.

(b) Verify that there is also equilibrium in the hamburger market and comment on the sources of the workers' total income.

(c) Suppose that the two firms each laid off 2 workers as Uchitelle wrote. Assume the laid-off workers get no income whatsoever. Also suppose that the remaining 28 workers receive the same wage as before. Show the situation on a labor market diagram. Are the firms' profits higher? What about the workers' incomes?

Answer to Review Problems:

5. *MBAs_a.*

(a) Since $y = 10000L^{1/2}$, $L(y) = \left(\frac{y}{10000}\right)^2$. Thus,

$$
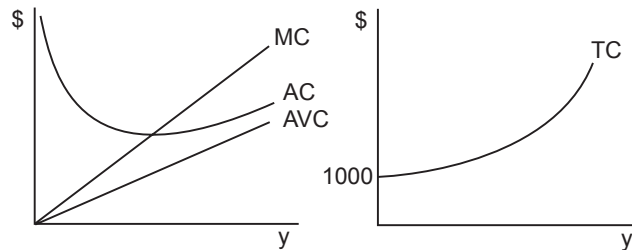\begin{aligned}
TC(y) &= 1000 + wL = 1000 + 100\left(\frac{y}{10000}\right)^2 \\
AC(y) &= \frac{1000}{y} + \frac{y}{1000^2} \\
AVC(y) &= \frac{y}{1000^2} \\
MC(y) &= \frac{y}{500000}
\end{aligned}
$$

(b)

(c) We know that a profit-maximizing, perfectly competitive firm sets $p = MC(y)$. Here, that implies

$$
\frac{y}{500000} = 2
$$

Solving this for y, we find that $y = 1,000,000$. Then $L(1,000,000) = 10,000$. Since there are 5 such firms, the total number hired is 50,000.

(d) Now the labor needed is:

$$
L(y) = \left(\frac{y}{10000}\right)^{300/149}
$$

and the optimal output solve:

$$MC(y) = 100 \frac{1}{10000}^{300/149} \frac{300}{149} y^{151/149} = 1.60$$

Now the solution is $y = 750,000$ and $L(750,000) = 5,959$, for a total market employment of 29,795.

6. *UncleKarl_a.*

   (a) Assuming you get paid the coupon at the end of the year, the present value equation is:

   $$1000 = \frac{50}{1+i} + \frac{1050}{1+i} \Rightarrow 1000(1+i) = 1100 \Rightarrow i = 10\%$$

   (b) We know that you can buy a risk-free bond and get a yield of 10%. Therefore any risk-free investment should have a cost of capital of $0.10 per dollar invested. Presumably the on-line music business is very risky, so a cost of capital of $0.15 would be more appropriate. (Indeed, a cost of capital of more like $0.40 might be reasonable.)

   (c) Since $f(K) = 10K^{9/10}$, you need $K(q) = \frac{q}{10}^{10/9}$ units of capital to produce output q. Since capital costs $0.15, and you have an additional fixed cost of $5000 that also comes out of capital, the cost curves are:

   $$TC(q) = 0.15 \left(5000 + \frac{q}{10}^{10/9} \right) = 750 + 0.0116q^{10/9}$$
   $$AC(q) = \frac{TC(q)}{q} = \frac{750}{q} + 0.0116q^{1/9}$$
   $$MC(q) = \frac{dTC(q)}{dq} = 0.0129q^{1/9}$$

   If you draw the graph exactly, it is a little strange because marginal cost is concave:
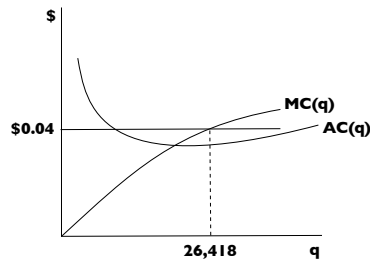
(d) Your profit maximizing quantity is where marginal cost equals price:

$$MC(q) = p \Rightarrow 0.0129q^{1/9} = 0.04 \Rightarrow q^* = 26,418$$

At that quantity, you need to invest $K(26,418) = 6,340$ dollars of capital plus the 5,000 dollar startup cost. Given that your cost of capital is $0.15, your total costs are $1,701. Your total revenue is $pq^* = 0.04 \times 26,418 = \$1,056.72$. Thus you actually lose money on this investment, since your revenues are lower than your costs, including the proper cost of capital. You should buy the bond instead!

7. *Lula.*

   (a) The formula to use here is

   $$P = \frac{A}{1+i} \quad R\$80 = \frac{R\$5 + R\$100}{1+i} \Rightarrow 1 + i = 1.3125 \Rightarrow i = 31.25\%$$

   (b) Investors perceived Lula as risky, and they demanded a higher risk premium on Brazilian government bonds. For an existing bond, the coupon and face value have already been set, so the only way for the yield to rise was for the present value to fall, as shown in the formula above.

8. *Deflate_a.* Total growth in real GDP between 1980 and 1990 was 36.5% and between 1990 and 2000 was 38.9%.

9. *Uchitelle_a.*

(a) The profit of one of the firms is $\pi(L) = p \cdot 25L^{1/4} - wL$. The first order condition for the optimal L to demand is
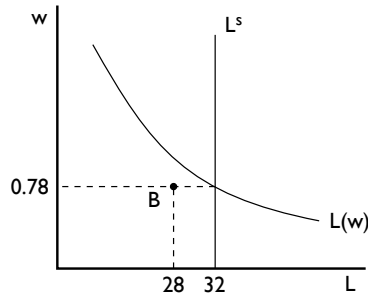
$$\frac{d\pi}{dL} = 6.25L^{-3/4} - w = 0$$

Solving for L, we find that the firm's labor demand is

$$L^D = 11.5w^{-4/3}$$

Setting labor supply equal to *market* labor demand gives us:

$$32 = 23w^{-4/3} \Rightarrow w^* = 0.78$$

(b) At this wage, each firm hires $L^D = 16$ workers and produces an output of $f(16) = 50$ hamburgers. Each firm makes a profit of $\pi(16) = 50 - 0.78 \cdot 16 = 37.52$. The income of the consumers is the total wage bill of $0.78 \cdot 32 = 25$ plus the dividends earned from owning the firms, for a total of $25 + 2 \cdot 37.5 = 100$. With nothing else to buy, this means consumers demand 100 hamburgers, which is the total output of the firms.

(c) Each firm now hires 14 workers, although this is not on their correctly-calculated labor demand curve as shown by point B in the diagram. Each firm's output is now $f(14) = 48.36$ hamburgers. The wage bill is only $0.78 \cdot 14 = 10.92$, so the profits of a firm are $\pi(14) = 48.36 - 10.92 = 37.44$. So firm

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profits fall slightly, which makes sense since they are no longer profit-maximizing. This implies that the dividend portion of household income also falls slightly.

The wage income portion of household income clearly falls, since fewer people are employed, although this reduction falls entirely on the 4 unemployed workers.