

ECON 110, Prof. Hogendorn

Problem Set 7

1. *SmallCountry*. Remember that a country's supply of loanable funds is the *net* supply after households that borrow are subtracted from those who save. Suppose there is a small country with 1000 households. 700 of these have a savings function $s = 50r$, where r is the rate of return on capital. The remaining 300 households have savings function $s = -1 + 10r$. (You can imagine that both the number of households and the amount of savings are in thousands.)
 - (a) Graph the individual and aggregate savings functions. Describe in words what happens to both types of household and the whole country when the interest rate rises from 3% to 11%.
 - (b) There are 100 firms, and each firm has an investment demand function $(i(r) = 10/r$. Find and graph the aggregate investment function for the whole country.
 - (c) Show that the equilibrium interest rate in this country is 16.6% (rounded to one decimal).
 - (d) In most countries, a real interest rate of 7% would be more typical. Do you think this country will have higher or lower economic growth than the typical country? Explain.
2. *OldGermans*. In Germany, the birth rate is low and the population is ageing. As a result, the working age population is falling at about 0.2% per year. It has been suggested that this population decline puts the German economy at risk. This question asks you to use our simple neoclassical model to evaluate that claim.

Let there be $\mathcal{L} = 243$ German workers who inelastically supply labor and who spend all of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. (Aggregate meaning we treat all the firms as if there were just 1.) There is no money, so the price of one beer is one beer.

- (a) Find the equilibrium real wage in the labor market and graph the labor market.
- (b) Verify that there is also equilibrium in the beer market and graph the production function. What share of workers' income comes from wages and what share from dividends?
- (c) Suppose that over 10 years, the German population falls and there are only $\mathcal{L}' = 198$ workers. Find the new general equilibrium.

3. *OldGermansSave*. As in OldGermans, there are 243 German workers who inelastically supply labor, but now they save 100 beers (in total) for the future and spend the rest of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. The German beer firms have aggregate investment demand of $I = 1200/r$, where r is the real interest rate.

- (a) Find the equilibrium real wage in the labor market and graph the labor market. Verify that there is also equilibrium in the beer market and graph the production function. (This just repeats OldGermans parts (a) and (b)).
- (b) Graph the capital market. What is the equilibrium real interest rate?
- (c) What happens if Germans become more pessimistic and start saving 110 beers?

(d) Go back to just 100 beers saved. What happens if the German government levies taxes of 60 beers but German Chancellor Angela Merkel drinks 80 beers?

4. *Botswana*. In his book *Globalization and its Discontents*, pg. 38, Joseph Stiglitz criticized the IMF's policy toward Botswana in 1981. He uses this as one example of a larger critique of the so-called "Washington Consensus" policy toward developing nations. This problem uses approximately accurate data to analyze the situation.

First, let's normalize Botswana's working population to $\mathcal{L} = 100$. Let Botswana have an economy-wide production function

$$Y = f(L) = 117.5L^{1/2}$$

and assume the firms represented by this function are owned by the workers. Set the price of Y equal to 1, and note that with the given production function, GDP at full employment is 1,175 million US dollars.

- (a) Find the equilibrium real wage in Botswana's labor market and graph the labor market. Graph the production function.
- (b) Now let's examine the capital market in 1980. Private Botswanans were saving 5% of GDP (assume perfectly inelastic with respect to the real interest rate). The government was spending 33% of GDP and collecting taxes of 34% of GDP. Firms' investment demand function was $I = 728 - 3520r$. Graph the domestic capital market and show the equilibrium real interest rate.
- (c) Actually, Botswana's real interest rate in 1980 was 10%, lower than what you found above. This was because Botswana could borrow in the world capital market. Redraw your capital market graph to show this lower real interest rate. How

much investment took place in Botswana? How large were foreign capital inflows?

- (d) Recall that total income from wages plus dividends has to equal consumption plus savings plus taxes. Find this for Botswana in 1980. Then recall that total output has to equal consumption plus investment plus government plus net exports. Find this for Botswana in 1980. Note: you just found consumption; investment and government were given in part (b) and (c); net exports is the residual that makes total income equal to total output.
- (e) Botswana faced two negative shocks in 1981 due to drought and problems in the diamond industry. We'll model this by saying that the production function changed for the worse to $Y = f(L) = 103.8L^{1/2}$. Show the new real wage.
- (f) The IMF advised Botswana to cut government spending, but it did not. Stiglitz agreed with Botswana's decision not to cut government spending. There are lots of reasons, but let's focus on just one: suppose some government spending is for healthcare, and that this spending creates a positive externality because it reduces disease throughout the population. Draw a graph of the healthcare market, and show that the private market would provide less than the socially optimal amount of healthcare.

Review Problems only, not to turn in:

- 5. *UncleKarl*. Your Uncle Karl gives you 20,000 dollars of capital.
 - (a) For \$1000, you can buy a risk-free government bond with a coupon of \$50 (payable at the end of the year), a face value of \$1050, and a maturity of one year. What is the yield on this bond?

- (b) Alternatively, you can invest some of the capital in a business venture providing downloadable music. For each dollar of capital invested over the course of one year, do you think it is more reasonable to let your cost of that capital be \$0.05, \$0.10, or \$0.15? Discuss your answer with reference to part (a), assuming you can buy fractional amounts of the bonds.
- (c) To simplify, assume no labor is involved in this business; the only factor is capital. Your production function is $f(K) = 100K^{9/10}$, where output is measured in the number of downloads. You must also use \$5,000 more of capital to pay a fixed cost to get started. What are the equations for your total, average, and marginal cost curves, using your answer to (b)? Graph the AC and MC curves.
- (d) If each download brings you revenue of \$0.04, how much capital should you invest in this business? Show this on your graph. Do you earn a competitive rate of return on your capital, or do you receive rents?
6. *Lula*. Suppose there is a Brazilian government bond with a face value of R\$100 (i.e. 100 reals, the currency of Brazil). The bond has a coupon of R\$5 and matures in 1 year.
- (a) If the bond's current price is R\$80, what is its yield?
- (b) Many investors thought that if Lula da Silva were elected president of Brazil, Brazilian debt would become more risky. Explain what probably happened to the price of Brazilian government bonds when Lula won.
7. *Deflate*. Given the information below about the U.S. economy, how much did real GDP grow between 1980 and 1990? Between 1990 and 2000?

	1980	1985	1990	1995	2000
Nominal GDP (trillions)	2.8	4.21	5.8	7.4	9.96
GDP deflator (1996=100)	57.0	73.7	86.5	98.1	106.9

8. *Uchitelle*. The following are quotes from an op-ed by Louis Uchitelle that appeared in the New York Times on August 25, 2002. It provides some food for thought, but we can evaluate the arguments a lot more clearly in a simple macroeconomic model. At the end of the article is a suggestion for trying to model Uchitelle's idea.

In Alice in Wonderland fashion, we talk of expansion and ignore the contraction all around us. We convince ourselves that out of cost-cutting will come prosperity. But while cost-cutting can lift a single company or two, when practiced widely enough it can pull down an economy. And that is happening today.

... consider what happens in an imaginary country where Burger King and McDonald's are the entire business sector and the total national output 100 hamburgers a day, evenly divided between the companies matches the demand from this nation's consumers. Demand and sales revenue, however, stay flat. So Burger King lays off two workers and uses the saved wages partly to fatten profits and partly to discount prices by just enough to take sales and revenue away from McDonald's. And McDonald's responds in kind. But soon, the four laid-off workers, with little income, buy fewer hamburgers, and the nation's total consumption drops to 95 hamburgers a day. That sets off another round of cost-cutting and price discounting, and our imaginary nation sinks gradually into stagnation or deep reces-

sion not unlike America in the 1930's.

Let the entire population of the economy be 32 workers who inelastically supply labor and who spend all of their income on hamburgers. Let McDonald's and Burger King be identical firms that each have production function $f(L) = 25L^{0.25}$. Let them both behave as perfect competitors. Let the price of one burger be one burger.

- (a) Find the equilibrium real wage in the labor market. Remember that there are TWO firms, so the total demand for labor is the sum of each firm's demand for labor. Illustrate with a graph.
- (b) Verify that there is also equilibrium in the hamburger market and comment on the sources of the workers' total income.
- (c) Suppose that the two firms each laid off 2 workers as Uchitelle wrote. Assume the laid-off workers get no income whatsoever. Also suppose that the remaining 28 workers receive the same wage as before. Show the situation on a labor market diagram. Are the firms' profits higher? What about the workers' incomes?

9. *Fear-goods*. This problem shows how in the neoclassical long-run macro model, widespread fear across an economy will not cause a recession! This is an important and comforting insight for the long run, but on the other hand, in the long run we are all dead...

Suppose the production function for the one representative firm in the economy is $Y = f(L) = 20L^{4/5}$. There are $\mathcal{L} = 40$ workers who inelastically supply labor.

- (a) Show that the labor demand curve is $L(w) = (16/w)^5$, graph the labor market, and show the equilibrium real wage.

- (b) Verify the national income accounts identity, i.e. that income from wages and dividends (which equals consumption) equals output (all of which is also consumption).
- (c) Now suppose that people in this country hear about the financial crisis. Everyone becomes very fearful of the future. The firm shifts down its labor demand curve to $L(w) = (8/w)^5$ – even though this is not profit maximizing because the production function remains unchanged. Assuming the labor market still clears, what happens to the wage, income from wages, income from dividends, and output?

Answer to Review Problems:

4. *UncleKarl_a.*

- (a) Assuming you get paid the coupon at the end of the year, the present value equation is:

$$1000 = \frac{50}{1+i} + \frac{1050}{1+i} \Rightarrow 1000(1+i) = 1100 \Rightarrow i = 10\%$$

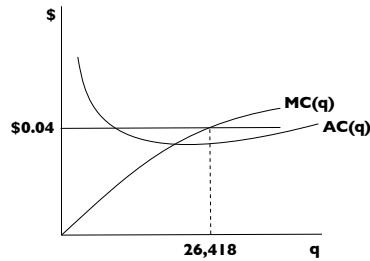
- (b) We know that you can buy a risk-free bond and get a yield of 10%. Therefore any risk-free investment should have a cost of capital of \$0.10 per dollar invested. Presumably the online music business is very risky, so a cost of capital of \$0.15 would be more appropriate. (Indeed, a cost of capital of more like \$0.40 might be reasonable.)
- (c) Since $f(K) = 10K^{9/10}$, you need $K(q) = \frac{q}{10}^{10/9}$ units of capital to produce output q . Since capital costs \$0.15, and you have an additional fixed cost of \$5000 that also comes out of capital, the cost curves are:

$$TC(q) = 0.15 \left(5000 + \frac{q}{10}^{10/9} \right) = 750 + 0.0116q^{10/9}$$

$$AC(q) = \frac{TC(q)}{q} = \frac{750}{q} + 0.0116q^{1/9}$$

$$MC(q) = \frac{dTC(q)}{dq} = 0.0129q^{1/9}$$

If you draw the graph exactly, it is a little strange because marginal cost is concave:



- (d) Your profit maximizing quantity is where marginal cost equals price:

$$MC(q) = p \Rightarrow 0.0129q^{1/9} = 0.04 \Rightarrow q^* = 26,418$$

At that quantity, you need to invest $K(26,418) = 6,340$ dollars of capital plus the 5,000 dollar startup cost. Given that your cost of capital is \$0.15, your total costs are \$1,701. Your total revenue is $pq^* = 0.04 \times 26,418 = \$1,056.72$. Thus you actually lose money on this investment, since your revenues are lower than your costs, including the proper cost of capital. You should buy the bond instead!

5. Lula.

- (a) The formula to use here is

$$P = \frac{A}{1+i} \quad R\$80 = \frac{R\$5 + R\$100}{1+i} \Rightarrow 1+i = 1.3125 \Rightarrow i = 31.25\%$$

- (b) Investors perceived Lula as risky, and they demanded a higher risk premium on Brazilian government bonds. For an existing bond, the coupon and face value have already been set, so the only way for the yield to rise was for the present value to fall, as shown in the formula above.

6. *Deflate_a*. Total growth in real GDP between 1980 and 1990 was 36.5% and between 1990 and 2000 was 38.9%.

7. *Uchitelle_a*.

(a) The profit of one of the firms is $\pi(L) = p \cdot 25L^{1/4} - wL$. The first order condition for the optimal L to demand is

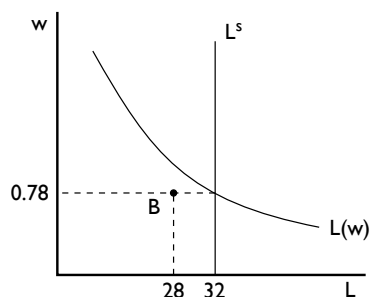
$$\frac{d\pi}{dL} = 6.25L^{-3/4} - w = 0$$

Solving for L , we find that the firm's labor demand is

$$L^D = 11.5w^{-4/3}$$

Setting labor supply equal to *market* labor demand gives us:

$$32 = 23w^{-4/3} \Rightarrow w^* = 0.78$$



(b) At this wage, each firm hires $L^D = 16$ workers and produces an output of $f(16) = 50$ hamburgers. Each firm makes a profit of $\pi(16) = 50 - 0.78 \cdot 16 = 37.52$. The income of the consumers is the total wage bill of $0.78 \cdot 32 = 25$ plus the dividends earned from owning the firms, for a total of $25 + 2 \cdot 37.5 = 100$. With nothing else to buy, this means consumers demand 100 hamburgers, which is the total output of the firms.

(c) Each firm now hires 14 workers, although this is not on their correctly-calculated labor demand curve as shown by point B in the diagram. Each firm's output is now $f(14) = 48.36$

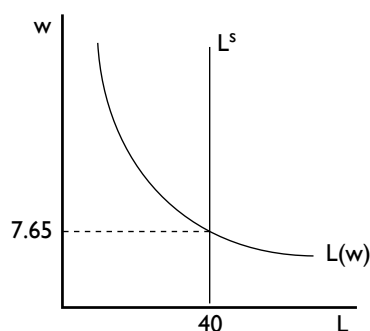
hamburgers. The wage bill is only $0.78 \cdot 14 = 10.92$, so the profits of a firm are $\pi(14) = 48.36 - 10.92 = 37.44$. So firm profits fall slightly, which makes sense since they are no longer profit-maximizing. This implies that the dividend portion of household income also falls slightly.

The wage income portion of household income clearly falls, since fewer people are employed, although this reduction falls entirely on the 4 unemployed workers.

8. *Fear-goods_a.*

- (a) Firms maximize profits by setting the marginal product of labor equal to the wage:

$$f'(L) = w \Rightarrow \frac{4}{5}20L^{-1/5} = w \Rightarrow L^{-1/5} = \frac{w}{16} \Rightarrow L(w) = \left(\frac{16}{w}\right)^5$$



- (b) Since the labor market clears, employment is 40, and output is $Y = f(40) = 382.5$.

Income is equal to wages plus dividends. Wages are $wL = 7.65 \cdot 40 = 306$. To find dividends, we need to find the profits of the firm:

$$\Pi = pq - wL = 1 \cdot 382.5 - 306 = 76.5$$

So total income is $306 + 76.5 = 382.5$ which does indeed equal output.

- (c) The labor market now equilibrates off the new, irrational labor demand curve, so

$$\left(\frac{8}{w}\right)^5 = 40 \Rightarrow w = 3.83$$

Not surprising, workers' wages fall because the firms' collective fear has essentially the same effect as if they all colluded to reduce wages. But since this is a neoclassical model, the labor market does still clear, and all 40 workers are still employed. That means that output is still $Y = f(40) = 382.5$. It remains to be seen if there is really income to pay for this output. Total wages are now only $3.83 \cdot 40 = 153.2$. But firm profits now rise (due to the lower labor costs) to $1 \cdot 382.5 - 153.2 = 229.3$. Therefore, dividends go up a lot, and total income is still $153.2 + 229.3 = 382.5$, exactly enough to equal output.