

ECON 110, Prof. Hogendorn

Problem Set 6

1. *Water.* The government has offered to give you a monopoly if you will provide water to a city. The inverse demand curve is $p(Q) = 1000 - 0.01Q$ and the average cost curve is $AC(Q) = \frac{25,000,000}{Q} + 100$.

- (a) What are the marginal revenue and marginal cost curves?
- (b) What is the optimal price you should charge and quantity you should produce? What is the profit of the monopolist?
- (c) Graph this situation carefully.
- (d) If the government were to give this firm a lump-sum subsidy, how big should it be if (1) the government is concerned only about paying the smallest possible subsidy or (2) it is concerned with overall welfare?

2. *Movie Windows.* The movie industry is struggling to adapt to technology change. Traditionally, a movie was released in cinemas for a 4-month “window,” and then it became available on DVD. Now DVD sales have fallen because consumers have more alternatives available for watching movies. Some movie studios are thinking about shortening the window (and sending films direct to Netflix, iTunes, etc.) in order to increase sales.

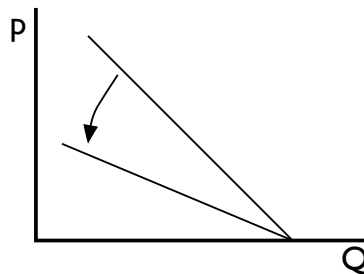
- (a) During the window, a movie studio has a monopoly on selling that particular movie to cinemas. Let the demand curve be

$$p = 1.64 - 0.034Q$$

and the marginal cost be a constant: $MC = AVC = 0.28$.

Write the profit function. Write out the first order condition, but do not solve for Q yet. Write in words the mathematical and the economic intuition behind the first order condition.

- (b) The monopoly price they set is $p_m = 0.96$, which is to say that the studio takes 96% of the cinema's box office revenue. The movie sells $Q_m = 20$ (million) tickets. Verify mathematically that these are the optimal monopoly price and quantity, and illustrate on a diagram.
- (c) Show in your diagram and calculate the amount of deadweight loss caused by the monopoly. (Note that using the "price" of 0.96, deadweight loss will be expressed as a percent of total ticket sales, not in dollars. To convert to dollars, just multiply by an average ticket price of \$8.)
- (d) True or false, and explain with a graph: since the monopoly price is well above the average variable cost, all movies make economic profits.
- (e) If the cinema window were reduced, waiting for the movie to come out on DVD or online would be a better substitute for impatient consumers, increasing elasticity but also shifting demand. Let's say that demand would pivot like this:



Show what happens on the monopoly diagram. Does monopoly price rise or fall? Monopoly operating profit? You don't have

to find any of these mathematically, but you do need to show them graphically.

3. *USChinaWages*. Suppose the production functions of a US and a Chinese textile mill are the same:

$$q = f(L) = -(L - 10)^2 + 100$$

Assume that neither mill ever hires more than 10 workers, and both factories are perfect competitors in both the textile and labor markets.

- (a) Graph the production function. Are there diminishing, constant, or increasing returns to labor?
 - (b) If the wage in China is \$0.57 and the wage in the United States is \$11, and the price per unit of output is \$1, how many workers will the Chinese mill hire? How many at the US mill?
 - (c) True or false, and explain: If the production function and wages are exactly as described here, it shows that the workers at the US textile mill are more skilled than the workers at the Chinese textile mill.
 - (d) Find the labor demand curve $L(w)$ for the factories. What is the elasticity of labor demanded with respect to the wage in the US? In China?
4. *Generators*. It is 2 in the afternoon on a hot day in July. Everyone in the city has their air conditioning turned up, with the result that the typical household demands 6 kWh (kilowatt hours) of electricity during the 2-3pm time slot. Their demand is perfectly inelastic because they are so hot, they don't care about the price of electricity!

There are two generating companies (GenCos) serving this city. Each one operates an oil-fired power plant that can produce electricity according to the production function

$$f(g) = (540g)^{\frac{1}{3}}$$

where g is gallons of oil and q is kWh of electricity per household in the city. The price of oil is 200¢ per gallon. Each GenCo also has a fixed cost of 20¢ per household. The price of electricity is p and the GenCos are price-takers.

- (a) The managers at GenCo A like to maximize their profits in terms of the quantity q of electricity per household that they produce. Write down the GenCo A profit function $\Pi(q)$. Derive GenCo A's supply curve.
- (b) The managers at GenCo B work differently. They figure out how much oil to buy to maximize profits. Write down the GenCo B profit function $\Pi(g)$. Derive GenCo B's supply curve (this will require an extra step relative to your answer for part a).
- (c) Describe in words why the two GenCos end up with the same supply curves.
- (d) What is the market equilibrium price of electricity? Draw a graph of the market equilibrium.
- (e) Derive and graph the marginal and average cost curves for one of the firms. At the market equilibrium price, calculate and label on the graph the profit or loss of the firm.
- (f) Describe what will happen in this market in the long run, and show the effects in both the market graph and the graph of an individual firm. Just show how the curves will shift; don't calculate the actual quantities and prices in the long run.

Review Problems only, not to turn in:

5. *EightFirms*. Suppose there are 8 firms supplying a given market. Each firm has the same total cost curve, which is

$$TC(q) = 20 + 12q + 2q^2$$

Each of the firms is a perfect competitor. *Market* demand is $q(p) = 60 - p$. What is the equilibrium price in this market? How much does each firm produce? Draw graphs to illustrate your answer.

6. *Long*. Derive and graph the long-run competitive equilibrium price associated with the following long-run total cost curve: $TC(q) = 1000 + 50q^2$.
7. *ChinaMobile*. This problem is loosely based on reality: Every year, cellular phone equipment becomes cheaper, and China Mobile's costs fall. Specifically, assume that in year 1, the marginal cost per minute is 0.20 yuan and in year 2 it falls to 0.10 yuan. (Note, in both years, MC is constant, i.e. horizontal.)
- Let demand (in minutes per typical consumer) be given by $Q(p) = 100 - 100p$. Treating China Mobile as a monopoly, what is the profit maximizing price and number of minutes in year 1? What about year 2? On the same graph, show the optima in both years.
 - Suppose that China Mobile committed to the quantities from (a) in year 1 and year 2, but that the demand estimate turned out to be a mistake. Really demand is $Q'(p) = 60 - 60p$. In terms of foregone profits, are China Mobile's problems getting worse or better over time?
 - From the point of view of China as a whole, was the mistake bad or good? In money terms, how much did China gain or lose in year 2? Illustrate on a graph.

(d) Just for fun: Who do you think China Mobile hired to do the initial demand estimate?

8. *Boomerangs*. Amherst Guy was fired from Barn & Company for his ridiculous advice to Dolty. Now he is managing a boomerang factory outside of Perth. The factory has a production function $q = f(v)$, where v is wood and q is boomerangs. The company is a price-taker in both the boomerang and wood markets.

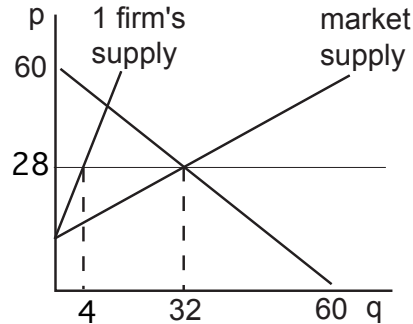
Amherst Guy says to the company's president, "You know I went to Amherst, so I actually know TWO ways to maximize profits. I could choose the optimal quantity of boomerangs by setting marginal cost equal to average cost, or I could choose the optimal amount of wood by setting the price of wood equal to the unemployment rate. Either way, I will get the same supply curve for our firm."

- (a) Correct AG's statement.
- (b) AG wanted to set marginal cost equal to average cost for his first method. Is there anything special about that point? Is it likely that the firm will end up doing this? Illustrate with a graph.
- (c) Suppose the Australian government decides that boomerang production causes a negative externality due to deforestation. Suppose that it decides to correct this externality using a Pigouvian tax. Show what will happen on a supply and demand diagram for the boomerang market. Show any dead-weight losses created or avoided by the tax.
- (d) If the explicit functional form of the production function is $q = 5v^{1/3}$, what is the boomerang maker's conditional demand for wood and what is its unconditional demand for wood? (Assume p is the price of boomerangs and p_v is the price of wood.)

Answers to Review Problems:

5. *Eight Firms*. Each firm has marginal cost curve $MC(q) = 12 + 4q$. Since each firm will optimally set price equal to marginal cost, we invert this curve and each firm has supply curve $s_i(p) = \frac{1}{4}p - 3$. Then market supply is eight times this, or $s(p) = 2p - 24$.

Market equilibrium occurs where supply equals demand, or $2p - 24 = 60 - p \Rightarrow 3p = 84 \Rightarrow p = 28$. Each firm produces $s_i(28) = \frac{1}{4}28 - 3 = 4$.



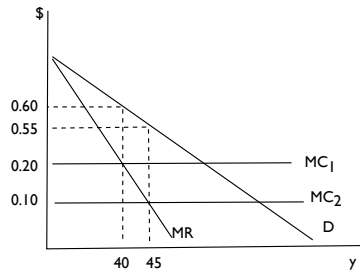
6. *Long_a*. In the long run, there will be entry if $p > AC$ and exit if $p < AC$. Therefore we are looking for a point where both $p = MC$ (short-run optimizing) and $p = AC$ (long-run equilibrium). The only such point is where:

$$\begin{aligned}
 MC(q) &= AC(q) \\
 100q &= \frac{1000}{q} + 50q \\
 50q &= \frac{1000}{q} \\
 q^2 &= 20 \\
 q &= 4.47 \\
 p = MC(4.47) &= 447
 \end{aligned}$$

7. *ChinaMobile_a.*

- (a) The inverse demand curve is $p(Q) = 1 - 0.01Q$. For a monopoly, profit is maximized when marginal revenue equals marginal cost. $TR = p(Q)Q = Q - 0.01Q^2$, so marginal revenue is $MR = 1 - 0.02Q$. Then in year 1 the profit maximizing quantity is $1 - 0.02Q = 0.2 \Rightarrow Q = 40$. The price at this quantity is $p(40) = 0.60$.

In year 2, the same calculations with the new marginal cost give $1 - 0.02Q = 0.1 \Rightarrow Q = 45$ and $p(45) = 0.55$. The graph looks like this:



- (b) The true inverse demand curve turns out to be $p'(Q) = 1 - 0.017Q$. In year 1, they mistakenly set $Q = 40$. This gives them a price of $p'(40) = 0.32$. Their profit is $\Pi = pQ - TC(Q) = 0.32 \times 40 - 0.2 \times 40 = 4.8$.

Actually, marginal revenue was $1 - 0.034Q$, so the correct monopoly quantity was $1 - 0.034Q = 0.2 \Rightarrow Q' = 23.5 \Rightarrow p'(23.5) = 0.6$. The profit would have been $\Pi' = 0.6 \times 23.5 - 0.2 \times 23.5 = 9.4$. Thus, China Mobile forewent $\Pi' - \Pi = 9.4 - 4.8 = 4.6$ profit.

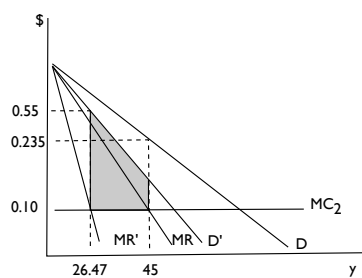
In year 2, they mistakenly set $Q = 45$. The price is $p'(45) = 0.235$. Their profit is $\Pi = pQ - TC(Q) = 0.235 \times 45 - 0.1 \times 45 = 6.075$.

Actually, the correct monopoly quantity was $1 - 0.034Q = 0.1 \Rightarrow Q' = 26.47 \Rightarrow p'(26.47) = 0.55$. The profit would

have been $\Pi' = 0.55 \times 26.47 - 0.1 \times 26.47 = 11.91$. Thus, China Mobile forewent $\Pi' - \Pi = 11.91 - 6.075 = 5.835$ profit.

Thus, not only did they lose a lot in both years (about half of potential profits), but things were worse in year 2 than in year 1. The reason is that there is more to lose when the monopoly has lower costs it can take advantage of.

- (c) Since monopolies inefficiently reduce quantities below the competitive level, and since price was still above marginal cost despite the mistake, we can be sure that China as a whole gained from the mistake. In year 2, 45 units were produced instead of 26.47. The added value (reduced deadweight loss) of these units was the area between the demand curve and the marginal cost curve between 26.47 and 45 units, shaded on the graph below.



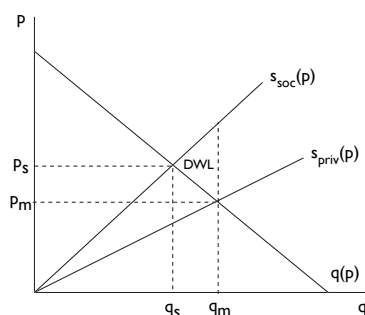
The numerical gain was:

$$\begin{aligned} \int_{26.47}^{45} 1 - 0.017Q - 0.2dQ &= \left| 0.8y - .0085Q^2 \right|_{26.47}^{45} \\ &= 18.7875 - 15.22 \\ &= 3.5675 \end{aligned}$$

(d) Amherst Guy!

8. *Boomerangs_a.*

- (a) “I could choose the optimal quantity of boomerangs by setting marginal cost equal to price of boomerangs, or I could choose the optimal amount of wood by setting the price of wood equal to the price of boomerangs times the marginal product of wood.
- (b) The point is special, because it must be the lowest point on the average cost curve. At lower quantities, marginal cost is lower than average cost and pulls it down, while at higher quantities the marginal cost is higher than average cost and pulls it up.
- The firm doesn't particularly like this point, since there are no economic profits, but in a competitive market, it can expect that entry of new firms will push price down until price equals average cost. So in long-run perfectly competitive equilibrium, the firm will probably end up producing at this point.
- (c) Marginal social cost is higher than marginal private cost due to deforestation. The market quantity q_m is too large relative to the social quantity q_s . The area marked DWL is the deadweight loss caused by the polluting over-production. If a Pigouvian Tax were enacted, it would shift the supply curve to the s_{soc} curve, resulting in a new price to consumers of p_s . The DWL would be avoided.



- (d) Conditional factor demand is how much wood to make q

boomerangs:

$$q = 5v^{1/3} \Rightarrow v^{1/3} = \frac{q}{5} \Rightarrow v(q) = \left(\frac{q}{5}\right)^3$$

To find the unconditional factor demand, we need to maximize profits first. The simplest way is the second method in part (a), set price of wood equal to the price of boomerangs times the marginal product of wood :

$$p_v = p \frac{dq(v)}{dv} \Rightarrow p_v = p \frac{5}{3} v^{-2/3} \Rightarrow v^{-2/3} = \frac{3p_v}{5p} \Rightarrow v(p_v) = \left(\frac{5p}{3p_v}\right)^{\frac{3}{2}}$$