ECON 110, Prof. Hogendorn

Problem Set 6 Answers

1. *Water_a.*

   (a) Total revenue and marginal revenue are:

$$\begin{aligned} TR(q) &= (1000 - 0.01q)q \\ MR(q) &= 1000 - 0.01q - 0.01q = 1000 - 0.02q \end{aligned}$$

   Total cost and marginal cost are:

$$\begin{aligned} TC(q) &= 25,000,000 + 100q \\ MC(q) &= 100 \end{aligned}$$

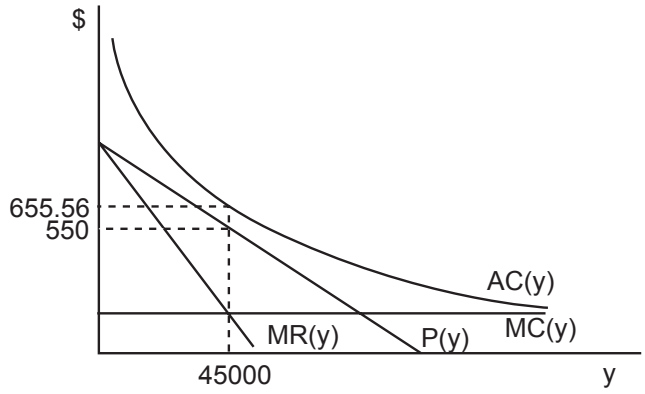
   (b) The optimal behavior is to set marginal revenue equal to marginal cost:

$$\begin{aligned} 1000 - 0.02q &= 100 \\ q &= 45000 \end{aligned}$$

   The price at this output is $p(45,000) = 1000 - 0.01 \cdot 45,000 = 550$. The profit is:

$$\begin{aligned} \Pi(45,000) &= (p(45,000) - AC(45,000))45,000 \\ &= (550 - 655.56)45,000 \\ &= -4,750,000 \end{aligned}$$

   (c) The key here is that the AC curve lies everywhere above the demand curve, so there's no way the monopoly can avoid a loss, even at the maximum "profit" level.



(d) If the government is only concerned with its own budget, the cheapest lump-sum subsidy needed is $4,750,000, which is just enough to offset the monopoly lost. We know this is the smallest possible lump-sum subsidy that will induce you to provide water service, because the monopoly output maximizes profits, or, in this case, minimizes losses.

If the government is concerned with overall welfare, it should induce the monopoly to set price equal to marginal cost. At this price, the monopolist makes $p(q) = 100 \Rightarrow q = 90,000$ units and its profits are:

$$\begin{aligned} \Pi(90,000) &= (p(90,000) - AC(90,000))90,000 \\ &= (100 - 377.78)90,000 \\ &= -25,000,000 \end{aligned}$$

Thus the government would have to provide a $25,000,000 subsidy on condition that the firm produces 90,000 units. This would be harder on the government budget, but it would maximize social welfare.

2. *MovieWindows.*

(a) Profits are

$$\Pi(\mathrm{Q}) = \mathrm{pQ} - \mathrm{TC}(\mathrm{Q}) = (1.64 - 0.034\mathrm{Q})\mathrm{Q} - 0.28\mathrm{Q}$$

The first order condition is

$$\frac{d\Pi(Q)}{dQ} = 1.64 - 0.034Q - 0.034Q - 0.28 = 0$$

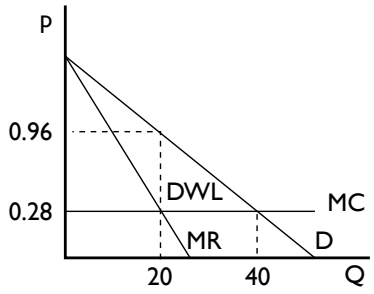
Mathematically, this finds the point of zero slope, the flat top of the "hill."

Economically, this finds where the marginal increase in revenue from selling one more unit equals the marginal increase in cost. After this, further production will reduce profits.

(b) Solving the first order condition gives

$$1.64 - 0.068Q = 0.28 \Rightarrow Q = 20$$

The graph shows the downward-sloping MR curve hitting the MC curve:

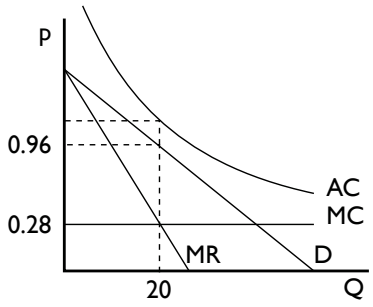


(c) If the studio behaved as a perfect competitor, it would rent the movie for 28% of ticket sales, and the quantity of tickets sold would be 40. So the deadweight loss is

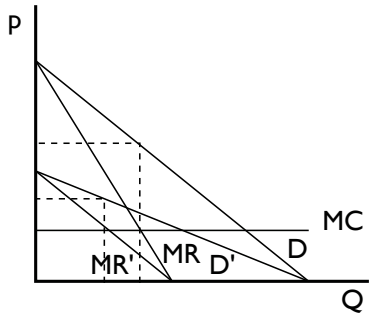
$$\frac{1}{2}(0.96 - 0.28)(40 - 20) = 6.8$$

Multiplying by \$8 suggests a deadweight loss of \$54.4 million for the movie.

(d) This is false. The movie described above earns an operating profit of $(0.96 - 0.28)20 = 13.6$, or about \$109 million when multiplied by the ticket price. But fixed costs might be higher than this, it depends on the movie. The movie shown in the following graph loses money on net.

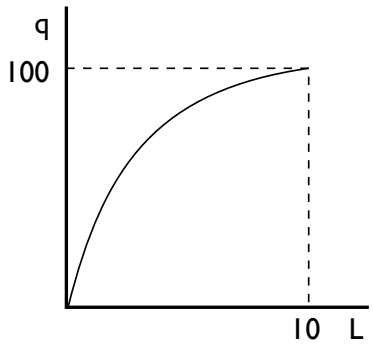


(e) The pivot shown in the graph will reduce both the monopoly price and the monopoly quantity, and therefore the monopoly operating profit.



3. *USChinaWages_a.*

(a) The derivative of the production function, i.e. the marginal product of labor, is $f'(L) = -2(L - 10)$. As long as $L < 10$, this is a postive number, so the production function slopes up. The second derivative is $f''(L) = -2$, which is negative, indicating that adding more labor decreases the marginal product. Hence, this is the case of diminishing returns to labor.



(b) Since both mills are perfect competitors, they will both set price times marginal product of labor equal to the wage. Using the formula for marginal product of labor from part (a), this is

$$-2(L-10) = w \Rightarrow L(w) = \frac{20-w}{2}$$

In China, we have $L(0.57) = 9.715$, while in the US we have $L(11) = 4.5$.

(c) False. Both mills have exactly the same production function, so for any given number of workers, the total and marginal product is the same at both mills. It is true that the marginal product of labor in the US is higher than in China, but this is because the wage is higher in the US, so profit maximization dictates that a mill there should hire fewer workers than in China. Since there are diminishing returns to labor, fewer workers means higher marginal product.

That said, it is true that in the real world, the production function is not the same in the US and China. US workers generally have more physical and human capital to work with, so in real life, US workers in most industries really are more productive than the same number of Chinese workers working in China. This is the main reason that wages are so much higher in the US.

(d) We already found the labor demand curve in part (b), it is

$L(w) = \frac{20-w}{2}$. Elasticity of labor demand with respect to the wage is defined as

$$\varepsilon = \frac{\%\Delta L}{\%\Delta w} = \frac{dL(w)}{dw}\frac{w}{L}$$

The derivative is $\frac{dL}{dw} = -½$. Thus, in the US the elasticity of labor demand to the wage is $-½\frac{11}{4.5} = -1.2$ and in China the elasticity is $-½\frac{0.57}{9.715} = -0.03$. It makes sense that US labor demand is so much more elastic because diminishing returns have not set in nearly as much, and thus marginal productivity is very sensitive to the number of workers hired.

4. *Generators_a.*

   (a) GenCo A's conditional factor demand for oil is found by inverting the production function:

   $$540g = q^3 \Rightarrow g(q) = \frac{q^3}{540}$$

   Profits are revenue minus fixed cost minus variable cost:

   $$\Pi(q) = pq - TC(q) = pq - 20 - 200\frac{q^3}{540}$$

   To maximize profits, take the derivative and set equal to 0. This is equivalent to the price-equals-marginal-cost condition.

   $$p - MC(q) = p - 600\frac{q^2}{540} = 0$$

   Finally, the supply curve is quantity as a function of price:

   $$q^2 = \frac{9}{10}p \Rightarrow s(p) = \left(\frac{9}{10}p\right)^{1/2}$$

   (b) With this method, costs are easy, just $20 + 200g$. Revenue is equal to price times the amount of production:

   $$\Pi(g) = pq(g) - 20 - 200g = p(540g)^{\frac{1}{3}} - 200g$$

To maximize profits, take the derivative and set equal to 0. This is equivalent to the price-times-marginal-product-equals-factor-price condition.

$$pMP_g - 200 = p\frac{1}{3}540^{\frac{1}{3}}g^{-\frac{2}{3}} - 200 = 0$$

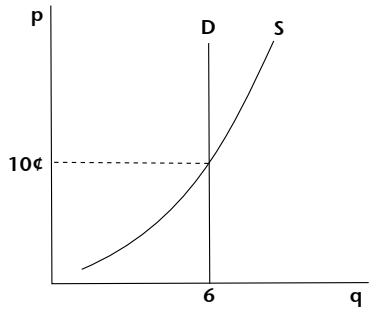
Now finding the supply curve takes two steps. First, find the unconditional factor demand $g(p)$, then use the production function to turn this into quantity produced as a function of price:

$$g^{-\frac{2}{3}} = \frac{600}{p540^{1/3}}$$
$$g(p) = \frac{600^{-3/2}}{p^{-3/2}540^{-1/2}}$$
$$s(p) = q(g(p)) = 540^{1/3}\frac{600^{-1/2}}{p^{-1/2}540^{-1/6}}$$
$$s(p) = 540^{1/2}\frac{600^{-1/2}}{p^{-1/2}540^{-1/6}}$$
$$s(p) = 540^{1/2}p^{1/2}600^{-1/2}$$

(c) Both gencos make their decisions on the basis of "should the company do a little more." GenCo A's condition says that additional electricity should be produced until the marginal cost of another unit equals the revenue from selling it. GenCo B's condition says that additional oil should be purchased until the cost of the oil equals the revenue generated from the marginal product (measured in electricity) made from the oil. These conditions are restatements of the same idea; both say "is it profitable to do a little more of this activity."

(d) Market equilibrium occurs when supply of electricity equals demand for electricity. Demand is just 6. Market supply is the sum of the supply curves of GenCos A and B. Thus,

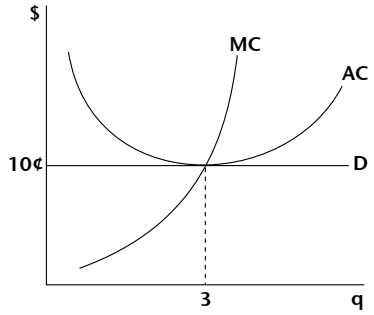
$$2s(p) = 2 \cdot 540^{1/2}p^{1/2}600^{-1/2} = 6$$
$$0.95p^{1/2} = 3$$
$$p = 10¢$$



(e) We already found TC above, so applying the definitions of AC and MC gives:

$$TC(q) = 20 + 200\frac{q^3}{540}$$
$$AC(q) = \frac{TC(q)}{q} = \frac{20}{q} + 200\frac{q^2}{540}$$
$$MC(q) = \frac{dTC(q)}{dq} = \frac{600}{540}q^2$$

To draw the graph, we need to find that $AC(3) = 10$.



Since the average cost is equal to price, profit is:

$$\Pi(3) = (p - AC(3))3 = (10 - 10)3 = 0$$

Thus, there is no super-normal profit or loss.

(f) The market is already in long-run equilibrium! Since the gencos make zero profits, they have no reason to leave the industry (their capital would earn the same return elsewhere). And if a new firm entered, it would shift market supply to S',

lowering the price and changing the demand facing an individual firm to D'. Then price would be below average cost, and the firm would make losses. Thus, no firm would enter.