

ECON 110, Prof. Hogendorn

Problem Set 7 Answers

1. *Campbell.*

- (a) Efforts like this improve the production function, creating more output at any level of labor input. Thus, the production function curve shifts up.
- (b) Suppose that Campbell is able to change its production function from $f(L) = 75L^{1/2}$ to $f(L) = 85L^{1/2}$. What is its labor demand curve for both cases?

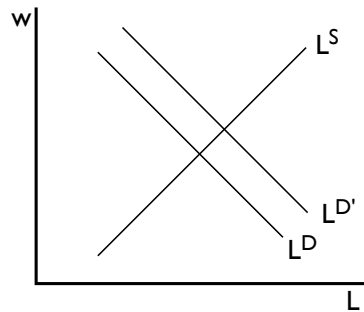
We have derived that the profit maximizing condition for a firm can be expressed as $pMP_L = w$. For these particular production functions this gives

$$p37.5L^{-1/2} = w \Rightarrow L^{1/2} = \frac{37.5p}{w} \Rightarrow L(w) = \frac{1406.25p^2}{w^2}$$

$$p42.5L^{-1/2} = w \Rightarrow L^{1/2} = \frac{42.5p}{w} \Rightarrow L(w) = \frac{1806.25p^2}{w^2}$$

- (c) Labor demand will be downward-sloping because at lower wages, firms are willing to let the marginal product of labor fall. This is true of Campbell and all the other employers. Assuming Campbell is large enough, the shift shown in part (b) may be apparent in the market demand curve as well.

Labor supply is likely to be upward-sloping. Although most people need to work no matter what, they don't need to work in Maxton, and people from out of town can come to work there.



- (d) We can see from the labor demand curves derived in part (b), and from the diagram, that labor demand actually shifts *up* from the productivity increases. If Campbell workers are now more productive, the incentive is to hire more, not less. On the other hand, if soup demand falls, then the price of soup would fall (or in real life, not rise enough to keep up with inflation). In this case, the p term in pMP_L would fall, and that really would decrease demand for labor. Thus, the second statement is a lot more likely to explain the change.
- (e) First we find variable cost using the production function:

$$Q = 85L^{1/2} \Rightarrow L(Q) = \frac{Q^2}{7,225} \Rightarrow wL(Q) = \frac{Q^2}{722.5}$$

Thus, marginal cost is

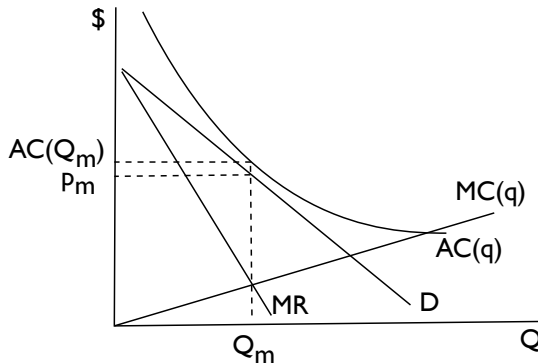
$$MC = \frac{Q}{361.25}$$

The total revenue of Campbell is $TR = p(Q)Q = (5 - 0.011Q)Q$
 so marginal revenue is $MR = 5 - 0.022Q$.

Then by setting marginal revenue equal to marginal cost we find that $Q_m = 201.87$ and plugging into the demand curve we get $p_m = 2.77$.

- (f) An average cost above even the monopoly price can certainly happen any time fixed costs are large enough. In this case,

Campbell still covers variable cost and the situation is probably temporary due to the recession, not permanent, so it should not shut down.



2. *Revolution_a.*

- (a) The profit function is total revenue minus total cost. To find the total cost, we need to know how much steel is used per bumper, which is just the inverse of the production function: $S(q) = (0.001q)^{12}$. Then:

$$\Pi(q) = 500q - 800(0.001q)^{12}$$

- (b) To maximize the profit function, take the derivative and set equal to 0:

$$\begin{aligned} \frac{d\Pi}{dq} &= 500 - 12 \cdot 800(0.001q)^{11} \cdot 0.001 = 0 \\ (0.001q)^{11} &= \frac{500}{12 \cdot 800 \cdot 0.001} \Rightarrow q^* = 1432 \end{aligned}$$

In words, the condition is price (or marginal revenue) equals marginal cost.

- (c) AG's method also starts with profits equal total revenue minus total cost, but they are all measured in terms of S . Since the amount of bumpers produced is always $q = 1000S^{\frac{1}{12}}$, AG's profit function is:

$$\Pi(S) = 500 \cdot 1000S^{\frac{1}{12}} - 800S$$

He also takes the derivative and sets equal to 0:

$$\frac{d\Pi}{dS} = \frac{1}{12} 500 \cdot 1000 S^{-\frac{11}{12}} - 800 = 0 \Rightarrow S^{-\frac{11}{12}} = \frac{800 \cdot 12}{500 \cdot 1000} \Rightarrow S = 74.6$$

In words, this condition is price of bumpers times the marginal product of steel equals the price of steel. If we evaluate the production function at AG's optimal S , we get $q = 1000(74.6)^{\frac{1}{12}} = 1432$, the same answer as in part (b).

- (d) AG's method is identical. Because the production function provides a direct relationship between S and y , either decision making process yields the same result. McCoy's condition says that additional output should be produced until the cost of another unit equals the revenue from selling it. Barn's condition says that additional steel should be purchased until the cost of the steel equals the revenue generated from selling bumpers made from the steel. These conditions are restatements of the same idea.

3. UncleKarl_a.

- (a) Assuming you get paid the coupon at the end of the year, the present value equation is:

$$1000 = \frac{50}{1+i} + \frac{1050}{1+i} \Rightarrow 1000(1+i) = 1100 \Rightarrow i = 10\%$$

- (b) We know that you can buy a risk-free bond and get a yield of 10%. Therefore any risk-free investment should have a cost of capital of \$0.10 per dollar invested. Presumably the on-line music business is very risky, so a cost of capital of \$0.15 would be more appropriate. (Indeed, a cost of capital of more like \$0.40 might be reasonable.)
- (c) Since $f(K) = 10K^{9/10}$, you need $K(q) = \frac{q}{10}^{10/9}$ units of capital to produce output q . Since capital costs \$0.15, and you

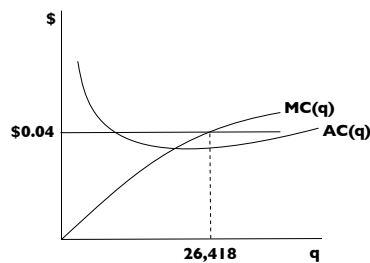
have an additional fixed cost of \$5000 that also comes out of capital, the cost curves are:

$$TC(q) = 0.15 \left(5000 + \frac{q}{10}^{10/9} \right) = 750 + 0.0116q^{10/9}$$

$$AC(q) = \frac{TC(q)}{q} = \frac{750}{q} + 0.0116q^{1/9}$$

$$MC(q) = \frac{dTC(q)}{dq} = 0.0129q^{1/9}$$

If you draw the graph exactly, it is a little strange because marginal cost is concave:



- (d) Your profit maximizing quantity is where marginal cost equals price:

$$MC(q) = p \Rightarrow 0.0129q^{1/9} = 0.04 \Rightarrow q^* = 26,418$$

At that quantity, you need to invest $K(26,418) = 6,340$ dollars of capital plus the 5,000 dollar startup cost. Given that your cost of capital is \$0.15, your total costs are \$1,701. Your total revenue is $pq^* = 0.04 \times 26,418 = \$1,056.72$. Thus you actually lose money on this investment, since your revenues are lower than your costs, including the proper cost of capital. You should buy the bond instead!