ECON 110, Professor Hogendorn

Problem Set 5

1. *FordToyota.* Let Ford and Toyota have two small factories, each with exactly the same production function for producing cars:

$$f(L) = 316L^{1/4}$$

Each company makes a single type of car that sells for a price of $p = \$25,000$. Each worker's annual salary is $\$62,500$. Each company makes 1000 cars per year at its factory.

   (a) What is the conditional factor demand for labor? What is the average variable cost and marginal cost of a car?

   (b) Toyota has a fixed cost of $15,000,000 at its factory. What is its operating profit and its net profit? Show the profits on a graph of price, average cost and average variable cost.

   (c) Ford has the same $15,000,000 fixed cost, plus additional fixed costs of $6,000,000 due to pensions for retired employees. What is its operating profit and its net profit? Show the profits on a graph of price, average cost, and average variable cost.

   (d) Assume production is fixed at 1000 cars and does not change from year to year. Toyota's factory will last for 5 years. Car prices and workers' salaries are both projected to grow at 5% per year. The production function will not change, and the same $15 million fixed cost occurs every year. The factory will have no value at all after 5 years. If the interest rate is 10%, how much is the factory worth today?

2. *MovieWindows.* The movie industry is struggling to adapt to technology change. Traditionally, a movie was released in cinemas for

a 4-month "window," and then it became available on DVD. Now DVD sales have fallen because consumers have more alternatives available for watching movies. Some movie studios are thinking about shortening the window (and sending films direct to Netflix, iTunes, etc.) in order to increase sales.

(a) During the window, a movie studio has a monopoly on selling that particular movie to cinemas. Let the demand curve be

$$p = 1.64 - 0.034Q$$

and the marginal cost be a constant: $MC = AVC = 0.28$.

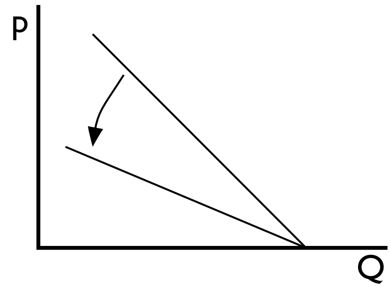
Write the profit function. Write out the first order condition, but do not solve for Q yet. Write in words the mathematical and the economic intuition behind the first order condition.

(b) The monopoly price they set is $p_m = 0.96$, which is to say that the studio takes 96% of the cinema's box office revenue. The movie sells $Q_m = 20$ (million) tickets. Verify mathematically that these are the optimal monopoly price and quantity, and illustrate on a diagram.

(c) Show in your diagram and calculate the amount of deadweight loss caused by the monopoly. (Note that using the "price" of 0.96, deadweight loss will be expressed as a percent of total ticket sales, not in dollars. To convert to dollars, just multiply by an average ticket price of $8.)

(d) True or false, and explain with a graph: since the monopoly price is well above the average variable cost, all movies make economic profits.

(e) If the cinema window were reduced, waiting for the movie to come out on DVD or online would be a better substitute for impatient consumers, increasing elasticity but also shifting demand. Let's say that demand would pivot like this:



Show what happens on the monopoly diagram. Does monopoly price rise or fall? Monopoly operating profit? You don't have to find any of these mathematically, but you do need to show them graphically.

3. *USChinaWages.* Suppose the production functions of a US and a Chinese textile mill are the same:

$$q = f(L) = -(L - 10)^2 + 100$$

Assume that neither mill ever hires more than 10 workers, and both factories are perfect competitors in both the textile and labor markets.

   (a) Graph the production function. Are there diminishing, constant, or increasing returns to labor?

   (b) If the wage in China is \$0.57 and the wage in the United States is \$11, and the price per unit of output is \$1, how many workers will the Chinese mill hire? How many at the US mill?

   (c) True or false, and explain: If the production function and wages are exactly as described here, it shows that the workers at the US textile mill are more skilled than the workers at the Chinese textile mill.

   (d) Find the labor demand curve $L(w)$ for the factories. What is the elasticity of labor demanded with resepct to the wage in the US? In China?

4. *Revolution.* After Wesleyan, you take a job with McCoy consulting. It was a tough decision because McCoy's big rival, Barn & Co., was also recruiting you. And now the pressure is on because you are making a big presentation to Dolty, an auto parts manufacturer which is a perfectly competitive firm.

   (a) The perfectly competitive price of a car bumper is \$500. Dolty uses steel to make bumpers according to the production function $f(S) = 1000S^{\frac{1}{12}}$ where S is tons of steel. The price of steel is \$800 per ton. What is Dolty's profit function $\Pi(q)$?

   (b) Describe the condition for profit maximization that shows how many bumpers Dolty should produce.

   (c) After you have shown the above, a team from Barn & Co. bursts into the room. Their young leader, known only by his initials A.G., says "Barn has a revolutionary new way to manage your firm. Don't think about bumpers, like these dinosaurs from McCoy! Instead, decide how much steel to buy!" He proceeds to write down a profit function $\Pi(S)$. Assuming he does this correctly, what does he write down? Show the condition for profit maximization using this function.

   (d) Now it's up to you to save McCoy's reputation. Argue (in words) that the profit maximization condition for A.G.'s method is exactly the same as the profit maximization condition in your method, and that Barn & Co. has no revolutionary management technique.

Review Problems only, not to turn in:

5. *Low.* Suppose a firm has cost curves $MC(q) = 0.0512q$ and $AC(q) = \frac{50}{q} + 0.0256q$. Use the first derivative of AC to prove that MC crosses AC at the lowest point on the AC curve.

6. *Nineteen.* A firm's production function is $q = f(L) = 10 + L^{1/3}$. The wage of labor is \$10. The firm has a fixed cost of \$47,500.

   (a) What are this firm's total, marginal, average, and average variable cost curves? (Hint: as a general rule, don't expand expressions like $(a+b)^c$ unless you really have to!)

   (b) Suppose the firm is a perfect competitor and the price of the good is \$3,000. How much profit does the firm make? How much labor is employed?

   (c) If the price fell by 19%, what would be the percentage change in profits and employment at this firm? Graph what happens in two ways: on a graph of the marginal and average cost curves and on a graph of the production function.

   (d) After the price falls, should the firm shut down?

7. *Generators.* It is 2 in the afternoon on a hot day in July. Everyone in the city has their air conditioning turned up, with the result that the typical household demands 6 kWh (kilowatt hours) of electricity during the 2-3pm time slot. Their demand is perfectly inelastic because they are so hot, they don't care about the price!

   There are two generating companies (GenCos) serving this city. Each one operates an oil-fired power plant that can produce electricity according to the production function

   $$f(g) = (540g)^{\frac{1}{3}}$$

   where g is gallons of oil and q is kWh of electricity per household in the city. The price of oil is 200¢ per gallon. Each GenCo also has a fixed cost of 20¢ per household. The price of electricity is p and the GenCos are price-takers.

   (a) The managers at GenCo A like to maximize their profits in terms of the quantity $q$of electricity per household that they produce. Write down the GenCo A profit function $\Pi(q)$. Derive GenCo A's supply curve.

(b) The managers at GenCo B work differently. They figure out how much oil to buy to maximize profits. Write down the GenCo B profit function $\Pi(g)$. Derive GenCo B's supply curve (this will require an extra step relative to your answer for (a)).

(c) Describe in words why the two GenCos end up with the same supply curves.

(d) What is the market equilibrium price of electricity? Draw a graph of the market equilibrium.

(e) Derive and graph the marginal and average cost curves for one of the firms. At the market equilibrium price, calculate and label on the graph the profit or loss of the firm.

(f) Describe what will happen in this market in the long run, and show the effects in both the market graph and the graph of an individual firm. Just show how the curves will shift; don't calculate the actual quantities and prices in the long run.

8. *MBAs*. The 2001 recession was very hard on the strategic consulting industry. Firms like McKinsey, Bain, and Booz Allen & Hamilton laid off 30% of their workforce.

   There were two components to the downturn. First, demand fell dramatically, in large part because of the demise of the dot-coms. Second, more executives began to have business school degrees and/or experience with the consulting firms. This made the "sage advice" of the consultants themselves less useful and effectively reduced the marginal product of laborers with MBA degrees (see *The Economist*, 11/2/02, pg. 61).

   For this problem, assume that the wage of MBAs is \$100. (Note: for more realism, you can think of all money amounts in this problem in thousands.)

(a) Let a typical consulting firm have production function $f(L) = 10000L^{1/2}$ and the firm also incurs a fixed cost of 1000. What is this firm's total cost function, average cost function, average variable cost function, and marginal cost function?

(b) Graph these curves.

(c) If the price of consulting is $p = 2$ and there are 5 consulting firms, how many MBAs are hired?

(d) Suppose that p falls to 1.60 and also the production function changes to $f(L) = 10000L^{149/300}$. Now how many MBAs are hired?

9. *Boomerangs.* Amherst Guy was fired from Barn & Company for his ridiculous advice to Dolty. Now he is managing a boomerang factory outside of Perth. The factory has a production function $q = f(v)$, where v is wood and q is boomerangs. The company is a price-taker in both the boomerang and wood markets.

   Amherst Guy says to the company's president, "You know I went to Amherst, so I actually know TWO ways to maximize profits. I could choose the optimal quantity of boomerangs by setting marginal cost equal to average cost, or I could choose the optimal amount of wood by setting the price of wood equal to the unemployment rate. Either way, I will get the same supply curve for our firm."

   (a) Correct AG's statement.

   (b) AG wanted to set marginal cost equal to average cost for his first method. Is there anything special about that point? Is it likely that the firm will end up doing this? Illustrate with a graph.

   (c) Suppose the Australian government decides that boomerang production causes a negative externality due to deforestation. Suppose that it decides to correct this externality using a Pigouvian tax. Show what will happen on a supply and demand diagram for the boomerang market. Show any deadweight losses created or avoided by the tax.

(d) If the explicit functional form of the production function is $q = 5v^{1/3}$, what is the boomerang maker's conditional demand for wood and what is its unconditional demand for wood? (Assume p is the price of boomerangs and p_v is the price of wood.)

Answers to Review Problems:

5. *Low_a.* At the lowest point on the AC curve, the slope is 0:

$$\frac{dAC}{dq} = -\frac{50}{q^2} + 0.0256 = 0 \Rightarrow q^2 = 1953.125 \Rightarrow q = 44.2$$

Setting MC=AC gives us

$$\frac{50}{q} + 0.0256q = 0.0512q \Rightarrow \frac{50}{q} = 0.0256q \Rightarrow q^2 = 1953.125 \Rightarrow q = 44.2$$

Either method gives the same answer.

6. *Nineteen_a.*

(a) Inverting the cost function gives $L = (q-10)^3$. Then the cost functions are:

$$\begin{aligned} TC(q) &= 47500 + wL = 47500 + 10(q-10)^3 \\ MC(q) &= 30(q-10)^2 \\ AC(q) &= \frac{47500}{q} + 10\frac{(q-10)^3}{q} \\ AVC(q) &= 10\frac{(q-10)^3}{q} \end{aligned}$$

(b) Write the profit function as $\pi(L)$ instead of $\pi(q)$:

$$\max_L \pi(L) = pq - TC(q) = 3000(10 + L^{1/3}) - 47500 - 10L$$

Then the first order condition is:

$$\frac{d\pi}{dL} = 1000L^{-2/3} - 10 = 0 \Rightarrow L = 1000$$

Profit is $\pi(1000) = 3000(10+10) - 47500 - 10000 = 2500$.

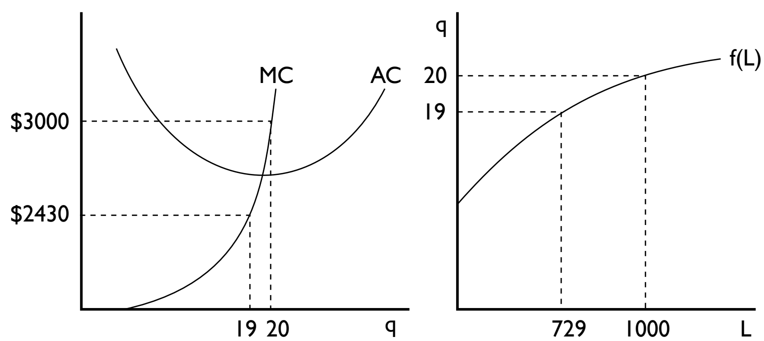
(c) The new first order condition would be

$$\frac{d\pi}{dL} = (1-0.19)1000L^{-2/3} - 10 = 0 \Rightarrow L = 729$$

The new profit is

$$\pi(729) = (1-.19)3000(10+9) - 47500 - 7290 = -8620$$

Thus, employment falls by 27.1% and profit falls by a whopping 445%!



(d) The new quantity is $f(729) = 19$, and $AVC(19) = 10\frac{(9)^3}{19} = 383.7$. This is less than the new price of $(1-0.19)3000 = 2430$. The firm should not shut down because it more than covers its variable costs, and in fact makes quite a large contribution to fixed costs. In the long run, however, it should shut down.

7. *Generators_a.*

(a) GenCo A's conditional factor demand for oil is found by inverting the production function:

$$540g = q^3 \Rightarrow g(q) = \frac{q^3}{540}$$

Profits are revenue minus fixed cost minus variable cost:

$$\Pi(q) = pq - TC(q) = pq - 20 - 200\frac{q^3}{540}$$

To maximize profits, take the derivative and set equal to 0. This is equivalent to the price-equals-marginal-cost condition.

$$\mathrm{p} - \mathrm{MC(q)} = \mathrm{p} - 600\frac{\mathrm{q}^2}{540} = 0$$

Finally, the supply curve is quantity as a function of price:

$$\mathrm{q}^2 = \frac{9}{10}\mathrm{p} \Rightarrow \mathrm{s(p)} = \left(\frac{9}{10}\mathrm{p}\right)^{1/2}$$

(b) With this method, costs are easy, just $20 + 200g$. Revenue is equal to price times the amount of production:

$$\Pi(g) = pq(g) - 20 - 200g = p(540g)^{\frac{1}{3}} - 200g$$

To maximize profits, take the derivative and set equal to 0. This is equivalent to the price-times-marginal-product-equals-factor-price condition.

$$pMP_g - 200 = p\frac{1}{3}540^{\frac{1}{3}}g^{-\frac{2}{3}} - 200 = 0$$

Now finding the supply curve takes two steps. First, find the unconditional factor demand $g(p)$, then use the production function to turn this into quantity produced as a function of price:

$$g^{-\frac{2}{3}} = \frac{600}{p540^{1/3}}$$

$$g(p) = \frac{600^{-3/2}}{p^{-3/2}540^{-1/2}}$$

$$s(p) = q(g(p)) = 540^{1/3}\frac{600^{-1/2}}{p^{-1/2}540^{-1/6}}$$

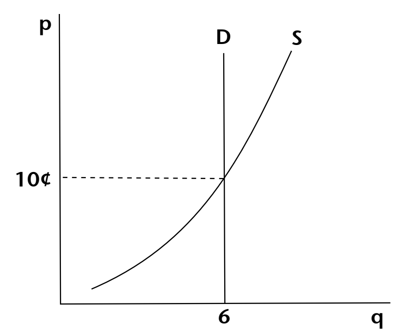
$$s(p) = 540^{1/2}\frac{600^{-1/2}}{p^{-1/2}540^{-1/6}}$$

$$s(p) = 540^{1/2}p^{1/2}600^{-1/2}$$

(c) Both gencos make their decisions on the basis of "should the company do a little more." GenCo A's condition says that additional electricity should be produced until the marginal cost of another unit equals the revenue from selling it. GenCo B's condition says that additional oil should be purchased until the cost of the oil equals the revenue generated from the marginal product (measured in electricity) made from the oil. These conditions are restatements of the same idea; both say "is it profitable to do a little more of this activity."

(d) Market equilibrium occurs when supply of electricity equals demand for electricity. Demand is just 6. Market supply is the sum of the supply curves of GenCos A and B. Thus,

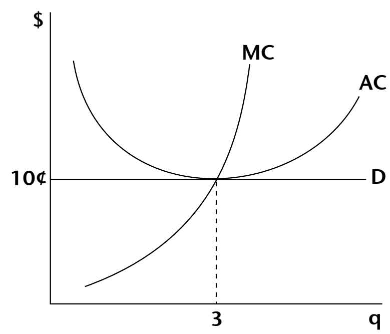
$$2s(p) = 2 \cdot 540^{1/2} p^{1/2} 600^{-1/2} = 6$$
$$0.95 p^{1/2} = 3$$
$$p = 10\acute{c}$$



(e) We already found TC above, so applying the definitions of AC and MC gives:

$$TC(q) = 20 + 200\frac{q^3}{540}$$
$$AC(q) = \frac{TC(q)}{q} = \frac{20}{q} + 200\frac{q^2}{540}$$
$$MC(q) = \frac{dTC(q)}{dq} = \frac{600}{540}q^2$$

To draw the graph, we need to find that $AC(3) = 10$.

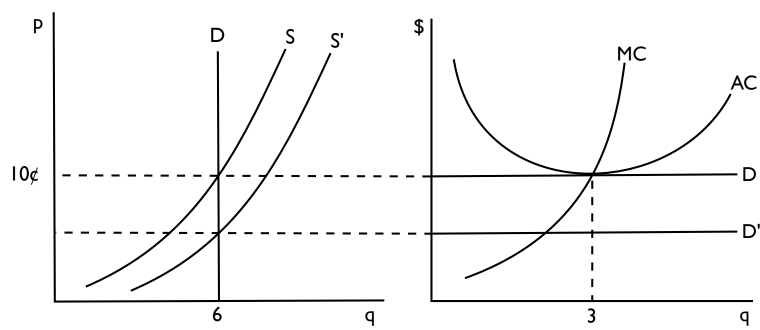


Since the average cost is equal to price, profit is:

$$\Pi(3) = (p - AC(3))3 = (10 - 10)3 = 0$$

Thus, there is no super-normal profit or loss.

(f) The market is already in long-run equilibrium! Since the gencos make zero profits, they have no reason to leave the industry (their capital would earn the same return elsewhere). And if a new firm entered, it would shift market supply to S', lowering the price and changing the demand facing an individual firm to D'. Then price would be below average cost, and the firm would make losses. Thus, no firm would enter.

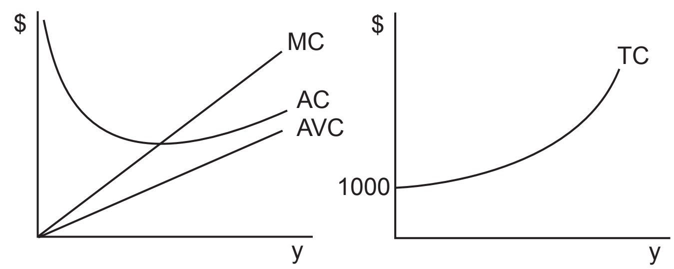


8. *MBAs_a.*

(a) Since $y = 10000L^{1/2}$, $L(y) = \left(\frac{y}{10000}\right)^2$. Thus,

$$\begin{aligned} TC(y) &= 1000 + wL = 1000 + 100\left(\frac{y}{10000}\right)^2 \\ AC(y) &= \frac{1000}{y} + \frac{y}{1000^2} \\ AVC(y) &= \frac{y}{1000^2} \\ MC(y) &= \frac{y}{500000} \end{aligned}$$

(b)



(c) We know that a profit-maximizing, perfectly competitive firm sets $p = MC(y)$. Here, that implies

$$\frac{y}{500000} = 2$$

Solving this for y, we find that $y = 1,000,000$. Then $L(1,000,000) = 10,000$. Since there are 5 such firms, the total number hired is 50,000.

(d) Now the labor needed is:

$$L(y) = \left(\frac{y}{10000}\right)^{300/149}$$

and the optimal output solve:

$$MC(y) = 100\frac{1}{10000}^{300/149}\frac{300}{149}y^{151/149} = 1.60$$

Now the solution is $y = 750,000$ and $L(750,000) = 5,959$, for a total market employment of 29,795.

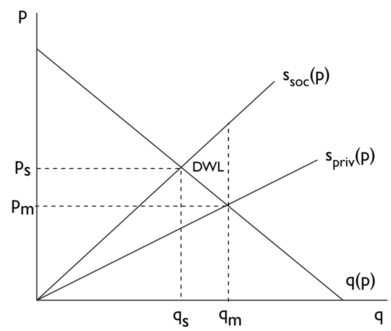
9. *Boomerangs_a.*

   (a) "I could choose the optimal quantity of boomerangs by setting marginal cost equal to price of boomerangs, or I could choose the optimal amount of wood by setting the price of wood equal to the price of boomerangs times the marginal product of wood.

   (b) The point is special, because it must be the lowest point on the average cost curve. At lower quantities, marginal cost is lower than average cost and pulls it down, while at higher quantities the marginal cost is higher than average cost and pulls it up.

   The firm doesn't particularly like this point, since there are no economic profits, but in a competitive market, it can expect that entry of new firms will push price down until price equals average cost. So in long-run perfectly competitive equilibrium, the firm will probably end up producing at this point.

   (c) Marginal social cost is higher than marginal private cost due to deforestation. The market quantity q_m is too large relative to the social quantity q_s. The area marked DWL is the deadweight loss caused by the polluting over-production. If a Pigouvian Tax were enacted, it would shift the supply curve to the s_{soc} curve, resulting in a new price to consumers of p_s. The DWL would be avoided.



(d) Conditional factor demand is how much wood to make q boomerangs:

$$q = 5v^{1/3} \Rightarrow v^{1/3} = \frac{q}{5} \Rightarrow v(q) = \left(\frac{q}{5}\right)^3$$

To find the unconditional factor demand, we need to maximize profits first. The simplest way is the second method in part (a), set price of wood equal to the price of boomerangs times the marginal product of wood :

$$p_v = p\frac{dq(v)}{dv} \Rightarrow p_v = p\frac{5}{3}v^{-2/3} \Rightarrow v^{-2/3} = \frac{3p_v}{5p} \Rightarrow v(p_v) = \left(\frac{5p}{3p_v}\right)^{\frac{3}{2}}$$