ECON 110, Professor Hogendorn, Spring 2019

# Second Midterm Exam Section 2: Answers

On corrected exams, I use x to mean wrong, check to mean right, ~ to mean partially right and partially wrong, … to mean more explanation needed, and ? to mean that it is not clear how you got a result (even if it's correct).

1. *Toys_a.*

    (a) The marginal and average cost are:

    $$MC(q) = \frac{dTC(q)}{dq} = 0.5q$$

    $$AC(q) = \frac{TC(q)}{q} = 0.25q + \frac{30}{q}$$

    A firm's supply curve is the marginal cost curve expressed as quantity as a function of price, so

    $$p = MC \Rightarrow p = 0.5q \Rightarrow q_s = 2p$$

    Five such firms will have market supply

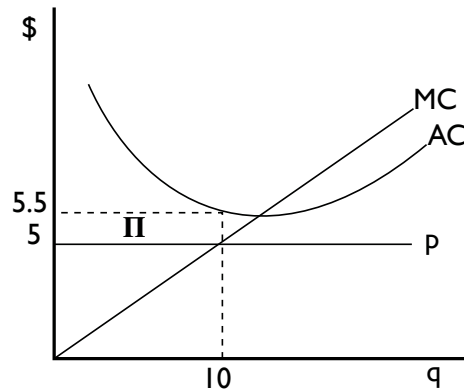    $$Q_s = 5q_s = 5 \times 2p = 10p$$

    Setting market demand equal to market supply gives us the equilibrium price

    $$Q_D = Q_S \Rightarrow 100 - 10p = 10p \Rightarrow p = \$5$$

    The individual firm thus produces 10 units, and has $AC(10) = 0.25 \times 10 + \frac{30}{10} = 5.5$. Profits are

    $$\Pi(10) = (P - AC(10))10 = (5 - 5.5)10 = -5$$

    So this firm is making losses. The graph of this situation, with profits labeled as $\Pi$, looks like

(b) Since the typical firm is making losses, we expect firms to leave the industry until prices rise enough to allow the remaining firms to break even. This happens when firms produce at the bottom of the AC curve, the point where $MC = AC$:

$$MC = AC \Rightarrow 0.5q = 0.25q + \frac{30}{q} \Rightarrow q = 11$$

At $q = 11$, $MC = 5.5$, so we need price to rise to 5.5. If there are $N$ firms, then market supply is $Q_s = 2Np$. Setting equal to market demand gives

$$Q_D = Q_S \Rightarrow 100 - 10p = 2Np \Rightarrow p = \frac{100}{2N + 10}$$

Since we need price to equal 5.5, there will be

$$5.5 = \frac{100}{2N + 10} \Rightarrow 11N + 55 = 100 \Rightarrow N = 4.09$$

firms. Since we can't have fractional firms, this means there will be 4 firms each earning a very small rent.

2. *UncleKarlPart1_a*. The budget line has a slope of $-1.06$ because money saved today grows to 1.06 times as much in the future. Based on that logic, the future value of the endowment is $FV = (1.06)20 = 21.2$.

The proposed consumption point involves saving $M_t - C_t = 10$ million dollars in the present, so there will be $(1.06)10 = 10.6$ million in the future. Thus the point lies below the budget line.
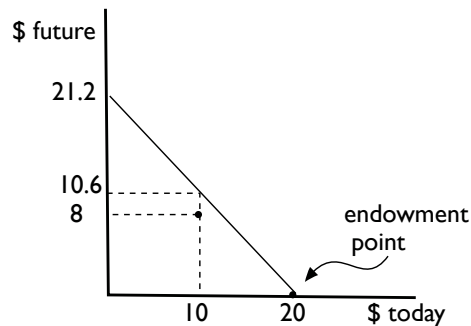


The point (10,8) cannot be utility-maximizing because it is possible to move up and to the left, the directions of higher utility. You could afford both more consumption today and more consumption in the future, so this can't be the best-possible point for you.

3. *UncleKarlPart2_a.*

   (a) The easiest way to find the optimal capital is to set up the profit function as a function of capital:

$$\Pi(K) = pf(K) - rK$$
$$= 0.04 \cdot 4000K^{3/5} - 0.15K$$

   Then the first order condition for a profit maximum sets the price times marginal product of capital equal to the cost of capital:

$$\frac{d\Pi(K)}{dK} = 96K^{-2/5} - 0.15 = 0$$

   Solving this for K gives $K^* = 10,362,151$.

At this optimal amount of capital, profits are

$$\pi(K^*) = 0.04 \cdot 4000(K^*)^{3/5} - 0.15K^* = 1,036,215$$

This is substantially more than 0, so you are earning rents, or a super-normal rate of return on your capital.

(b) This is bad advice because your Internet startup company is very risky, much riskier than a diversified portfolio of stocks. The risk-adjusted cost of capital for a business like this might well be 15%, just like the reputable investment banker said. If you nonetheless took the advice of AG, you would greatly increase the amount of capital you put into the business, making it much bigger. You would also count yourself as having a much bigger profit since your revenue would be higher and your cost of capital would be lower. But all of this would be based on an incorrect risk assessment, and if evaluated at the correct 15% rate, your bigger business would actually make lower profits than in part (a).