ECON 110, Professor Hogendorn, Spring 2019

Second Midterm Exam Section 3: Answers

On corrected exams, I use x to mean wrong, check to mean right, ~ to mean partially right and partially wrong, ... to mean more explanation needed, and ? to mean that it is not clear how you got a result (even if it's correct).

1. *Toys_a.*

   (a) The marginal and average cost are:

$$MC(q) = \frac{dTC(q)}{dq} = 0.5q$$
$$AC(q) = \frac{TC(q)}{q} = 0.25q + \frac{20}{q}$$

   A firm's supply curve is the marginal cost curve expressed as quantity as a function of price, so

$$p = MC \Rightarrow p = 0.5q \Rightarrow q_s = 2p$$

   Five such firms will have market supply

$$Q_s = 5q_s = 5 \times 2p = 10p$$

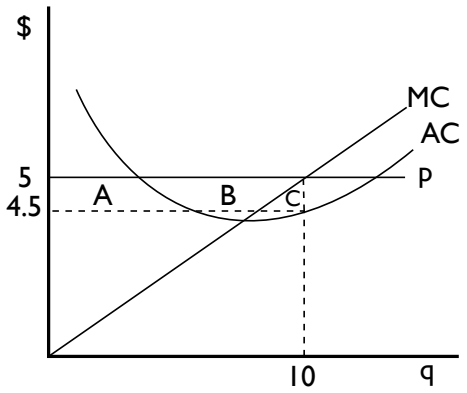
   Setting market demand equal to market supply gives us the equilibrium price

$$Q_D = Q_S \Rightarrow 100 - 10p = 10p \Rightarrow p = \$5$$

   The individual firm thus produces 10 units, and has $AC(10) = 0.25 \times 10 + \frac{20}{10} = 4.5$. Profits are

$$\Pi(10) = (P - AC(10))10 = (5 - 4.5)10 = 5$$

   So this firm is making net profits. The graph of this situation, with profits labeled as $A + B + C$, looks like



(b) Since the typical firm is making profits, we expect firms to enter the industry until prices fall enough that all firms break even. This happens when firms produce at the bottom of the AC curve, the point where $MC = AC$:

$$MC = AC \Rightarrow 0.5q = 0.25q + \frac{20}{q} \Rightarrow q = 8.9$$

At $q = 8.9$, $MC = 4.45$, so we need price to fall to 4.45. If there are N firms, then market supply is $Q_s = 2Np$. Setting equal to market demand gives

$$Q_D = Q_S \Rightarrow 100 - 10p = 2Np \Rightarrow p = \frac{100}{2N+10}$$

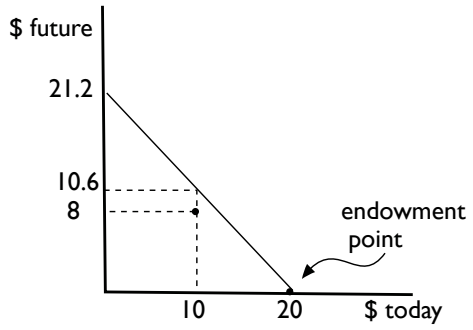
Since we need price to equal 4.45, there will be

$$4.45 = \frac{100}{2N+10} \Rightarrow 8.9N + 44.5 = 100 \Rightarrow N = 6.2$$

firms. Since we can't have fractional firms, this means there will be 6 firms each earning a small rent.

2. *UncleKarlPart1_a.* The budget line has a slope of -1.06 because money saved today grows to 1.06 times as much in the future. Based on that logic, the future value of the endowment is $FV = (1.06)20 = 21.2$.

The proposed consumption point involves saving $M_t - C_t = 10$ million dollars in the present, so there will be $(1.06)10 = 10.6$ million in the future. Thus the point lies below the budget line.



The point (10,8) cannot be utility-maximizing because it is possible to move up and to the left, the directions of higher utility. You could afford both more consumption today and more consumption in the future, so this can't be the best-possible point for you.

3. *UncleKarlPart2_a.*

   (a) The easiest way to find the optimal capital is to set up the profit function as a function of capital:

$$\begin{aligned}\Pi(K) &= pf(K) - rK \\ &= 0.04 \cdot 4000K^{3/5} - 0.15K\end{aligned}$$

   Then the first order condition for a profit maximum sets the price times marginal product of capital equal to the cost of capital:

$$\frac{d\Pi(K)}{dK} = 96K^{-2/5} - 0.15 = 0$$

   Solving this for K gives $K^* = 10,362,151$.

At this optimal amount of capital, profits are

$$\pi(K^*) = 0.04 \cdot 4000(K^*)^{3/5} - 0.15K^* = 1,036,215$$

This is substantially more than 0, so you are earning rents, or a super-normal rate of return on your capital.

(b) This is bad advice because your Internet startup company has already been evaluated to have a risk-adjusted cost of capital of 15%. Any further return you make above that is a rent, lucky you, and not a further risk that requires a higher return. If you nonetheless took the advice of AG, you would greatly decrease the amount of capital you put into the business, making it much smaller. You would also count yourself as having a lower profit since your revenue would be smaller and your cost of capital would be higher. Even if you evaluated this smaller business using the correct 15% rate, it would make lower profits than in part (a) because it is too small.