ECON 110, Professor Hogendorn

Problem Set 4

1. *Lawns.* Grass lawns create a variety of negative externalities, including air and noise pollution from mowing, herbicide and pesticide pollution, water scarcity from irrigation, and destruction of woody plants and shrubs that provide better wildlife habitat and carbon sequestration. The average American household spends around \$1200 per year on lawn care (obviously this varies enormously by household, but that's the average). Again using an average, there is about 1/3 acre of lawn per household (lawns are America's biggest and most polluting agricultural "crop").

   (a) Use the data point of price equals \$1.2 thousands and quantity equals 0.33, and suppose that the (private) price elasticity of demand for lawn is $E_d = -1.5$. What is a back-of-the-envelope linear demand curve for lawns? (Let $p = 1.2$, this problem is easier in thousands.)

   (b) Let the supply curve for lawns (really for lawn care products and services) be $Q_s(p) = 0.25 + 0.067p$. What is the equilibrium price, quantity, consumer, and producer surplus from lawns?

   (c) Suppose that the negative externalities from lawn *consumption* add up to \$400 per acre. What is the social demand curve $Q_{soc}(p)$?

   (d) What is the social equilibrium? How much deadweight loss is there? Calculate numerically and show on a graph.

   (e) If the government administered a Pigouvian tax by making each household pay \$400 per acre of lawn, how much tax revenue would be generated? Calculate numerically and show on a graph.

2. *CoalNaturalGas.* There are two local electricity markets, one called Amherst which uses coal and one called Middletown which uses natural gas.

   Both markets face the same demand curve, which is perfectly elastic at a price of \$90 per MWh (megawatt-hour).

   Amherst has an upward-sloping marginal private cost curve given by $MPC(Q) = 10 + 0.1Q$.

   Middletown has an upward-sloping marginal private cost curve of $MPC(Q) = 25 + 0.1Q$.

   (a) Draw two supply-and-demand diagrams for the two markets.

   (b) In Middletown, there are \$25 in external costs per MWh due to carbon dioxide (global warming) emissions. Find Middletown's marginal social cost curve and its socially optimal amount of production.

   (c) In Amherst there are \$50 in external costs per MWh due to carbon dioxide (global warming) emissions. Use the difference in externalities and costs to show graphically whether Amherst's socially optimal production is larger or smaller than Middletown's.

   (d) Suppose the government levied a tax of \$50 per MWh on the electricity production in *both* markets. Would this be socially optimal in these two markets? Would it create or remove deadweight loss? Illustrate your answer on two diagrams.

3. *FordToyota.* Let Ford and Toyota have two small factories, each with exactly the same production function for producing cars:

$$f(L) = 316L^{1/4}$$

Each company makes a single type of car that sells for a price of $p = \$25,000$. Each worker's annual salary is $\$62,500$. Each company makes 1000 cars per year at its factory.

(a) What is the conditional factor demand for labor? What is the average variable cost and marginal cost of a car?

(b) Toyota has a fixed cost of $15,000,000 at its factory. What is its operating profit and its net profit? Show the profits on a graph of price, average cost and average variable cost.

(c) Ford has the same $15,000,000 fixed cost, plus additional fixed costs of $6,000,000 due to pensions for retired employees. What is its operating profit and its net profit? Show the profits on a graph of price, average cost, and average variable cost.

(d) Assume production is fixed at 1000 cars and does not change from year to year. Toyota's factory will last for 5 years. Car prices and workers' salaries are both projected to grow at 5% per year. The production function will not change, and the same $15 million fixed cost occurs every year. The factory will have no residual value at all after 5 years. If the interest rate is 10%, how much is the factory worth today? (Assume the revenues and costs of the first year occur immediately and therefore are not discounted.)

4. *EightFirms*. Suppose there are 8 firms supplying a given market. Each firm has the same total cost curve, which is

$$\mathrm{TC(q)} = 20 + 12\mathrm{q} + 2\mathrm{q}^2$$

Each of the firms is a perfect competitor. *Market* demand is $q(p) = 60 - p$. What is the equilibrium price in this market? How much does each firm produce? Draw graphs to illustrate your answer.

5. *Long.* Derive and graph the long-run competitive equilibrium price associated with the following long-run total cost curve: $TC(q) = 1000 + 50q^2$.

Review Problems only, not to turn in:

6. *Fatburgers.* There are 400 fatburger consumers and 100 fatburger producers. The price of a fatburger, p, is measured in cents. Each of the 400 consumers has demand curve

$$Q_i(p) = 100 - \frac{p}{4}$$

Each producer has supply curve

$$S_i(p) = 4(p - 5)$$

(a) Determine the market supply and demand, find the equilibrium price, and draw on a graph.

(b) The government imposes a per-unit sales tax of t cents per fatburger. Find the new equilibrium price and quantity as a function of t.

(c) Show that the government achieves the maximum possible tax revenue when it sets $t = 197.5$ cents. You will need to find and maximize the government's revenue as a function of t. Remember that to maximize a function, you look for where the derivative equals 0.

(d) How much does the tax in part (c) reduce consumer surplus and producer surplus, and how much deadweight loss does it cause? Show on a graph as well as giving numerical results.

(e) You have just learned that when people eat fatburgers, it causes significant long-term health problems. Much of the cost of these health problems is paid for by the government rather than the individuals. In fact, careful analysis suggests that the government ends up paying \$1.975 in health costs for every fatburger eaten. Show how this information changes the graphical analysis of part (d). (Numerical results are not necessary.)

7. *SiliconValley.* In Silicon Valley, there are many information technology (IT) firms clustered in one place. This is usually attributed to positive externalities in production: when firm produces a product, the skilled workers can exchange ideas with one another, with venture capitalists, and so on. Thus, firms in Silicon Valley are more productive than similar firms elsewhere.

   (a) Graph the supply and demand curves for one IT good (e.g. web servers) in Silicon Valley. Show the positive externality in production.
   (b) Label the graph to show the external benefits and the deadweight loss in both the free-market and the socially optimal situations.
   (c) If the California government were to intervene in this market, what should it do?

8. *USChinaWages.* Suppose the production functions of a US and a Chinese textile mill are the same:

$$q = f(L) = -(L-10)^2 + 100$$

   Assume that neither mill ever hires more than 10 workers, and both factories are perfect competitors in both the textile and labor markets.

   (a) Graph the production function. Are there diminishing, constant, or increasing returns to labor?

(b) If the wage in China is \$0.57 and the wage in the United States is \$11, and the price per unit of output is \$1, how many workers will the Chinese mill hire? How many at the US mill?

(c) True or false, and explain: If the production function and wages are exactly as described here, it shows that the workers at the US textile mill are more skilled than the workers at the Chinese textile mill.

(d) Find the labor demand curve $L(w)$ for the factories. What is the elasticity of labor demanded with resepct to the wage in the US? In China?

9. *Low.* Suppose a firm has cost curves $MC(q) = 0.0512q$ and $AC(q) = \frac{50}{q} + 0.0256q$. Use the first derivative of AC to prove that MC crosses AC at the lowest point on the AC curve.

10. *MBAs.* The 2001 recession was very hard on the strategic consulting industry. Firms like McKinsey, Bain, and Booz Allen & Hamilton laid off 30% of their workforce.

    There were two components to the downturn. First, demand fell dramatically, in large part because of the demise of the dot-coms. Second, more executives began to have business school degrees and/or experience with the consulting firms. This made the "sage advice" of the consultants themselves less useful and effectively reduced the marginal product of laborers with MBA degrees (see *The Economist*, 11/2/02, pg. 61).

    For this problem, assume that the wage of MBAs is \$100. (Note: for more realism, you can think of all money amounts in this problem in thousands.)

    (a) Let a typical consulting firm have production function $f(L) = 10000L^{1/2}$ and the firm also incurs a fixed cost of 1000. What is this firm's total cost function, average cost function, average variable cost function, and marginal cost function?

(b) Graph these curves.

(c) If the price of consulting is $p = 2$ and there are 5 consulting firms, how many MBAs are hired?

(d) Suppose that p falls to 1.60 and also the production function changes to $f(L) = 10000L^{149/300}$. Now how many MBAs are hired?

11. *Nineteen.* A firm's production function is $q = f(L) = 10 + L^{1/3}$. The wage of labor is \$10. The firm has a fixed cost of \$47,500.

(a) What are this firm's total, marginal, average, and average variable cost curves? (Hint: as a general rule, don't expand expressions like $(a+b)^c$ unless you really have to!)

(b) Suppose the firm is a perfect competitor and the price of the good is \$3,000. How much profit does the firm make? How much labor is employed?

(c) If the price fell by 19%, what would be the percentage change in profits and employment at this firm? Graph what happens in two ways: on a graph of the marginal and average cost curves and on a graph of the production function.

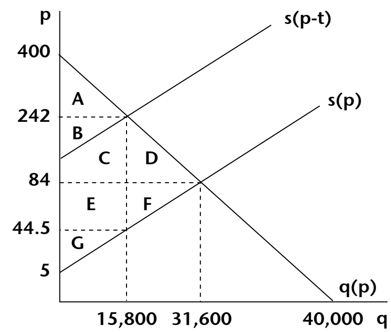
(d) After the price falls, should the firm shut down?

Answers to Review Problems:

5. *Fatburgers_a.*

(a) Market demand: $Q(p) = 400q_i(p) = 40,000 - 100p$. Market supply: $S(p) = 100s_i(p) = 400(p-5)$.

$$\begin{aligned} 40,000 - 100p &= 400(p-5) \\ 42,000 &= 500p \\ p &= 84 \\ q(84) &= 31,600 \end{aligned}$$



(b) This is a sales tax, so it is paid by producers and thus shifts the supply curve to $S(p-t)$ in the diagram. The new equilibrium price and quantity is found as follows:

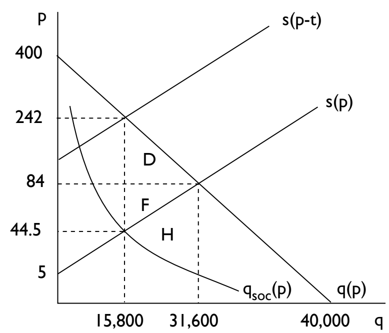
$$\begin{aligned}
40,000 - 100p &= 400(p - t - 5) \\
42,000 &= 500p - 400t \\
p(t) &= 84 + \frac{4}{5}t \\
Q(p(t)) &= 31,600 - 80t
\end{aligned}$$

(c) The government's revenue function is $R(t) = tQ(p(t)) = 31,600t - 80t^2$. We can maximize this function by taking the derivative and setting equal to 0:

$$\frac{dR(t)}{dt} = 31,600 - 160t = 0 \Rightarrow t^* = 197.5$$

(d) First, using the formulas from (b) we can find that $p(197.5) = 242$ and $q(p(197.5)) = 15,800$. Then in the graph, we have the following:
$\Delta CS = -B - C - D$
$= -(242-84)15,800 - \frac{1}{2}(242-84)(31,600-15,800) = -3,744,600.$
$\Delta PS = -E - F$
$= -(84-44.5)15,800 - \frac{1}{2}(84-44.5)(31,600-15,800) = -936,150$
$DWL = D + F = \frac{1}{2}(242 - 44.5)(31,600 - 15,800) = 1,560,250$

(e) This is a very tricky question! There is actually a negative externality in *consumption* of fatburgers. That means that the social benefit is less than the demand curve. But we don't actually know anything about the shape of the Q_{soc} curve, perhaps it is some nonlinear curve like in the diagram below. All that we know is that at the $Q = 15,800$ point, the negative externality is exactly equal to the sales tax.

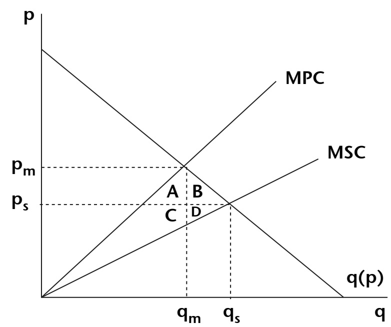


Without the tax, there would be a deadweight loss of area H. There would be too much consumption, and the costs $S(p)$ would exceed the benefits Q_{soc}.

The sales tax corrects for the externality perfectly at the $Q = 15,800$ point. It is not a true Pigouvian tax in the sense that if there were any shifts in the supply curve, it would no longer be optimal. But the supposed deadweight loss of $D + F$ that we found in part (d) turns out not to be a deadweight loss at all. Instead, it turns out that it was private consumer and and producer surplus that was exactly offset by the negative health externality.

6. *SiliconValley_a.*

(a)



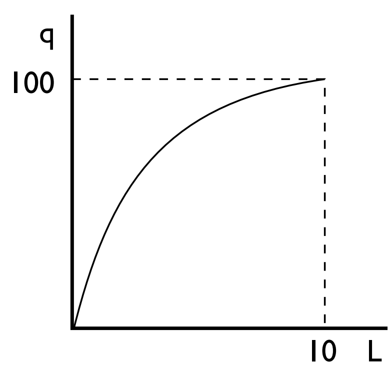
(b) At the free market equilibrium, external benefits are $A+C$, and there is a deadweight loss $B+D$.

At the social optimum, external benefits are $A+B+C+D$.

(c) It could provide a subsidy so that the price of output fell to p_s in the graph. This would increase quantity demanded to q_s and correct for the externality.

7. *USChinaWages_a.*

(a) The derivative of the production function, i.e. the marginal product of labor, is $MP_L = \frac{df}{dL} = -2(L-10)$. As long as $L < 10$, this is a postive number, so the production function slopes up. The second derivative is $\frac{d^2 f}{dL^2} = -2$, which is negative, indicating that adding more labor decreases the marginal product. Hence, this is the case of diminishing returns to labor.



(b) Since both mills are perfect competitors, they will both set price equal to marginal cost. The conditional factor demand is found by solving $q = f(L)$ for L:

$$\begin{aligned} q &= -(L-10)^2 + 100 \\ (L-10)^2 &= 100 - q \\ -(L-10) &= (100-q)^{1/2} \\ L(q) &= 10 - (100-q)^{1/2} \end{aligned}$$

(Notice that we took the *negative* square root of $(L - 10^2)$ because we know that $L - 10$ is a negative number.)

Thus, $TVC(q) = w\left[10 - (100-q)^{1/2}\right]$. Marginal cost is the derivative of total cost, and since fixed cost is fixed, we can also think of the marginal cost as the derviatvie of total variable cost. Hence,

$$MC(q) = \frac{dTVC(q)}{dq} = w \times -\frac{1}{2}(100-q)^{-\frac{1}{2}}(-1) = \frac{w}{2(100-q)^{\frac{1}{2}}}$$

Setting marginal cost equal to price gives

$$MC(q) = 1 \Rightarrow \frac{w}{2(100-q)^{\frac{1}{2}}} = 1 \Rightarrow (100-q)^{\frac{1}{2}} = \frac{w}{2} \Rightarrow$$

$$100 - q = \frac{w^2}{4} \Rightarrow q^* = 100 - \frac{w^2}{4}$$

So the American factory sets $q^* = 100 - \frac{11^2}{4} = 69.75$ and the Chinese factory sets $q^* = 100 - \frac{0.57^2}{4} = 99.92$.

If we substitute q^* in to the conditional factor demand, we get

$$L(q^*) = 10 - \left(100 - (100 - \frac{w^2}{4})\right)^{1/2} = 10 - \left(\frac{w^2}{4}\right)^{1/2} = 10 - \frac{w}{2}$$

as the labor demand curve. Subbing in the wages gives the amount of labor hired in both factories:

$$10 - \frac{11}{2}4.5 \qquad 10 - \frac{0.57}{2} = 9.7$$

This all would have been easier if we had just used the marginal product of labor from part (a). Then setting $pMP_L = w$ gives

$$1 \times -2(L-10) = w \Rightarrow L(w) = 10 - \frac{w}{2}$$

In China, we have $L(0.57) = 9.715$, while in the US we have $L(11) = 4.5$.

(c) False. Both mills have exactly the same production function, so for any given number of workers, the total and marginal product is the same at both mills. It is true that the marginal product of labor in the US is higher than in China, but this is because the wage is higher in the US, so profit maximization dictates that a mill there should hire fewer workers than in China. Since there are diminishing returns to labor, fewer workers means higher marginal product.

That said, it is true that in the real world, the production function is not the same in the US and China. US workers generally have more physical and human capital to work with, so in real life, US workers in most industries really are more productive than the same number of Chinese workers working in China. This is the main reason that wages are so much higher in the US.

(d) We already found the labor demand curve in part (b), it is $L(w) = 10 - \frac{w}{2}$. Elasticity of labor demand with respect to the wage is defined as

$$\epsilon = \frac{\%\Delta L}{\%\Delta w} = \frac{dL(w)}{dw}\frac{w}{L}$$

The derivative is $\frac{dL}{dw} = -\frac{1}{2}$. Thus, in the US the elasticity of labor demand to the wage is $-\frac{1}{2}\frac{11}{4.5} = -1.2$ and in China the elasticity is $-\frac{1}{2}\frac{0.57}{9.715} = -0.03$. It makes sense that US labor demand is so much more elastic because diminishing returns have not set in nearly as much, and thus marginal productivity is very sensitive to the number of workers hired.

8. *Low_a*. At the lowest point on the AC curve, the slope is 0:

$$\frac{dAC}{dq} = -\frac{50}{q^2} + 0.0256 = 0 \Rightarrow q^2 = 1953.125 \Rightarrow q = 44.2$$

Setting MC=AC gives us

$$\frac{50}{q} + 0.0256q = 0.0512q \Rightarrow \frac{50}{q} = 0.0256q \Rightarrow q^2 = 1953.125 \Rightarrow q = 44.2$$

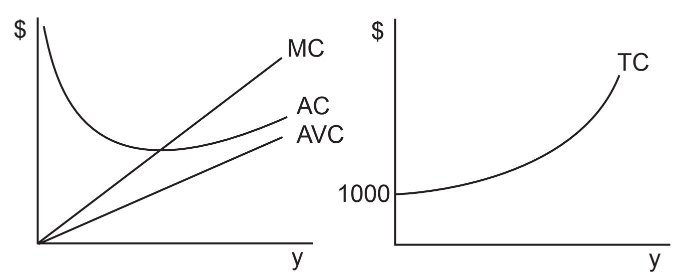
Either method gives the same answer.

9. *MBAs_a*.

(a) Since $y = 10000L^{1/2}$, $L(y) = \left(\frac{y}{10000}\right)^2$. Thus,

$$\begin{aligned} TC(y) &= 1000 + wL = 1000 + 100\left(\frac{y}{10000}\right)^2 \\ AC(y) &= \frac{1000}{y} + \frac{y}{1000^2} \\ AVC(y) &= \frac{y}{1000^2} \\ MC(y) &= \frac{y}{500000} \end{aligned}$$

(b)



(c) We know that a profit-maximizing, perfectly competitive firm sets $p = MC(y)$. Here, that implies

$$\frac{y}{500000} = 2$$

Solving this for y, we find that $y = 1,000,000$. Then $L(1,000,000) = 10,000$. Since there are 5 such firms, the total number hired is 50,000.

(d) Now the labor needed is:

$$L(y) = \left(\frac{y}{10000}\right)^{300/149}$$

and the optimal output solve:

$$MC(y) = 100\frac{1}{10000}^{300/149}\frac{300}{149}y^{151/149} = 1.60$$

Now the solution is $y = 750,000$ and $L(750,000) = 5,959$, for a total market employment of 29,795.

10. *Nineteen_a.*

(a) Inverting the cost function gives $L = (q-10)^3$. Then the cost functions are:

$$\begin{aligned}
TC(q) &= 47500 + wL = 47500 + 10(q-10)^3 \\
MC(q) &= 30(q-10)^2 \\
AC(q) &= \frac{47500}{q} + 10\frac{(q-10)^3}{q} \\
AVC(q) &= 10\frac{(q-10)^3}{q}
\end{aligned}$$

(b) Write the profit function as $\pi(L)$ instead of $\pi(q)$:

$$\max_L \pi(L) = pq - TC(q) = 3000(10 + L^{1/3}) - 47500 - 10L$$

Then the first order condition is:

$$\frac{d\pi}{dL} = 1000L^{-2/3} - 10 = 0 \Rightarrow L = 1000$$

Profit is $\pi(1000) = 3000(10+10) - 47500 - 10000 = 2500$.

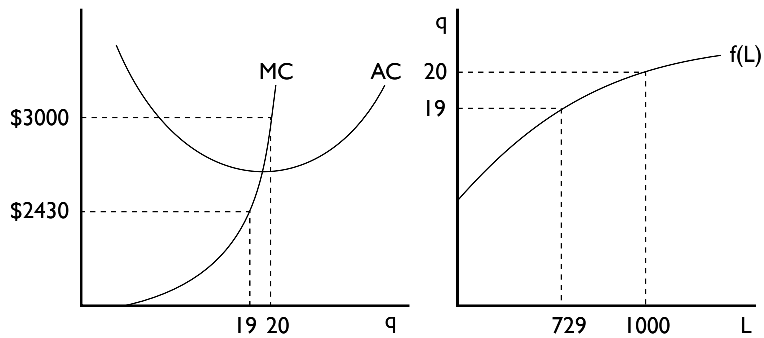
(c) The new first order condition would be

$$\frac{d\pi}{dL} = (1-0.19)1000L^{-2/3} - 10 = 0 \Rightarrow L = 729$$

The new profit is

$$\pi(729) = (1-.19)3000(10+9) - 47500 - 7290 = -8620$$

Thus, employment falls by 27.1% and profit falls by a whopping 445%!



(d) The new quantity is $f(729) = 19$, and $AVC(19) = 10\frac{(9)^3}{19} = 383.7$. This is less than the new price of $(1-0.19)3000 = 2430$. The firm should not shut down because it more than covers its variable costs, and in fact makes quite a large contribution to fixed costs. In the long run, however, it should shut down.