ECON 110, Professor Hogendorn

Problem Set 4 Answers

1. *Lawns_*.

   (a) Writing the formula for elasticity on a linear demand curve and working backwards gives:

   $$E_d = -1.5 = -b\frac{p}{q} = -b\frac{1.2}{0.33} \Rightarrow b = 0.4125$$

   Now that we know the slope, the intercept can be found by making sure the line goes through the point we identified:

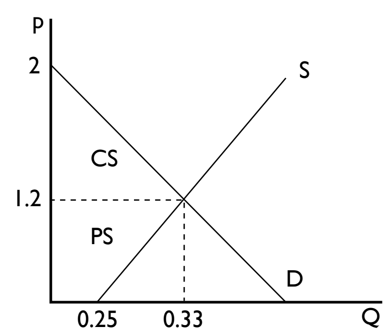
   $$Q_d = a - bp \Rightarrow 0.33 = a - 0.4125 \times 1.2 = 0.33 \Rightarrow a = 0.825$$

   Thus, the demand curve is $Q_d(p) = 0.825 - 0.4125p$.

   (b) Setting demand equal to supply gives

   $$0.825 - 0.4125p = 0.25 + 0.067p \Rightarrow 0.575 = 0.4795p \Rightarrow p = 1.20$$

   The corresponding quantity, using the demand curve, is $Q_d(1.2) = 0.33$. (Not surprisingly, we picked this point as the equilibrium on purpose.) To find consumer and producer surplus, it's probably easiest to draw the graph, finding the choke price and the supply intercept:



Then the area of CS is $\frac{1}{2}(2-1.2)0.33 = 0.132$. The area of PS involves a rectangle and a triangle added together:

$$.25 \times 1.2 + \frac{1}{2} \times 1.2 \times (0.33 - 0.25) = 0.348.$$

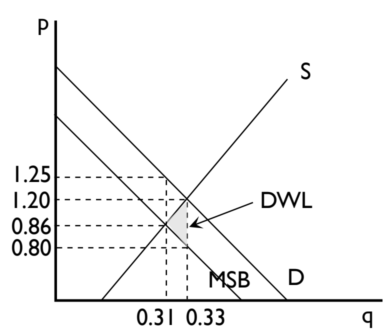
(c) The private demand curve is $Q_d(p) = 0.825 - 0.4125p$. If we invert it, we get the marginal private benefits of a quantity q of lawn: $MPC = 2 - 2.4Q$. But now it turns out that there is a \$400 social cost that needs to be subtracted off, so we subtract 0.4 from the above: $MSB = 1.6 - 2.4Q$. To go back to the non-inverse demand curve, just invert again:

$$\begin{aligned} p &= 1.6 - 2.4Q \\ 2.4Q &= 1.6 - p \\ Q_{soc} &= 0.67 - 0.42p \end{aligned}$$

(d) Setting supply equal to the social demand we just found,

$$\begin{aligned} Q_s &= Q_{soc} \\ 0.25 + 0.067p &= 0.67 - 0.42p \\ 0.49p &= 0.42 \\ p_{soc} &= 0.86 \end{aligned}$$

The corresponding quantity, using the supply curve, is $Q_s(0.86) = 0.25 + 0.067 \times 0.86 = 0.31$. To find deadweight loss it's easiest to draw the graph.



The area of the deadweight loss is
$\frac{1}{2}(1.2-0.80)(0.33-0.31) = 0.004$, or about \$4 per household.

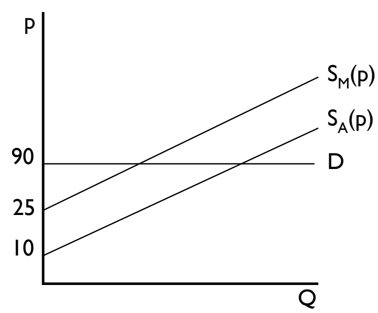
(e) Since the tax would be levied directly on the household, it would shift the private demand curve exactly to the social demand curve. The private demand curve at $q = 0.31$ gives a price of 1.25, so the tax revenue would be $(1.25-0.86)0.31 =$ or \$0.1209 thousands, i.e. \$120.90 for the average household.

2. *CoalNaturalGas_a.*

(a) These are perfectly competitive firms, so they both set $MC(Q) = p$ to achieve a profit maximum. Thus, we can just invert the two marginal cost curves to get the supply curves:

$$\begin{array}{cc} 10+0.1Q = p & 25+0.1Q = p \\ Q = \frac{p-10}{0.1} & Q = \frac{p-25}{0.1} \\ s_A(p) = -100+10p & s_M(p) = -250+10p \end{array}$$

(b) The supply and demand diagram looks like this:



(c) Add the \$25 to Middletown's marginal private cost curve, then set equal to price to find the supply curve as in (a). So:

$$MSC(Q) = 25+0.1Q+25$$

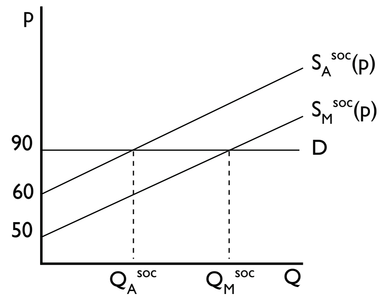
$$p = 50+0.1Q$$

$$Q = \frac{p-50}{0.1}$$

$$s_M^{soc} = -500+10p$$

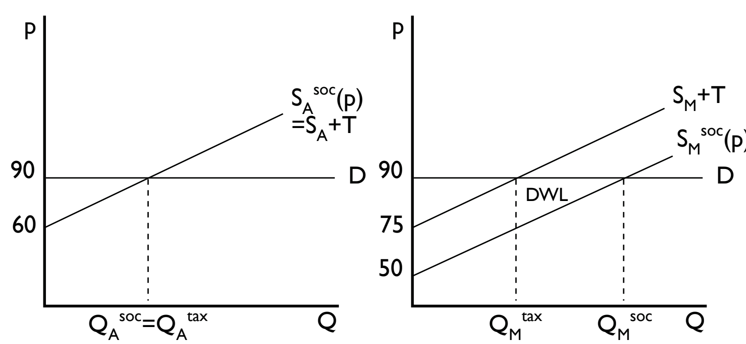
At the market price of \$90, the social supply is $s_M^{soc}(90) = 400$.

(d) We know that the MPC curve of Amherst is \$15 lower than the MPC of Middletown. But we also know that the externality is \$25 more. Thus, the MSC of Amherst must be \$10 higher than for Middletown, which means the socially optimal production will be smaller.



(e) The tax would be exactly the same as the externality in Amherst, so the new "S+T" curve would exactly coincide with Amherst's social supply curve. Thus, it is socially optimal for Amherst.

But the new S+T curve for Middletown is shifted up too much, \$25 too much to be precise. Thus the new quantity produced in Middletown is too low relative to the social optimum. Some electricity that has social benefits (of \$90) greater than social costs go unproduced. This creates DWL.



3. *FordToyota_a.*

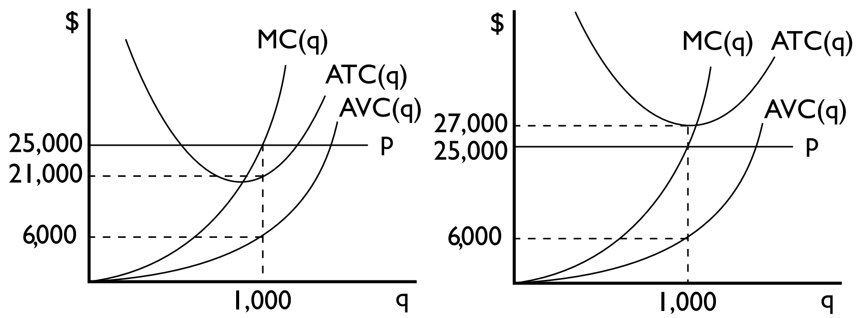
(a) By inverting the production function, we find that $L(q) = \left(\frac{q}{316}\right)^4$.

Then total variable cost is $TVC(q) = 0.000006q^4$, average variable cost is $AVC(q) = 0.000006q^3$, and marginal cost is the derivative of total variable cost, or $MC(q) = 0.000025q^3$.

(b) Operating profit is

$$\pi_T = (p - AVC(1000))1000 = (25000 - 6000)1000 = 19,000,000$$

Net profit is $\Pi_T = \pi_T - F = 19,000,000 - 15,000,000 = 4,000,000$. The graph looks something like the left panel below:



(c) Ford has precisely the same variable costs, so it also has the same marginal cost, average variable cost, and operating profit. The only difference is the net profit, which is exactly \$6,000,000 less, or $\pi_F = -2,000,000$. The graph is like the right panel above.

(d) In year 1, the net profit is 4,000,000 as we saw in part (b), and it does not need to be discounted because it's in "current" money. In year 2, the operating profit will grow by 5%, since both its components, revenue and variable cost, grow by 5%. But it must be discounted by 10% so the year 2 operating profit is $\frac{1.05}{1.10}\pi_T$ and the year 2 net profit is $\frac{1.05\pi_T - F}{1.10}$. Continuing this pattern, the discounted present value formula is:

$$PV = \pi_T - F + \frac{1.05\pi_T - F}{1.10} + \frac{1.05^2\pi_T - F}{1.10^2} + \frac{1.05^3\pi_T - F}{1.10^3} + \frac{1.05^4\pi_T - F}{1.10^4}$$

Looking at this equation, it makes more sense to split each term into two parts, one for π_T and one for F. For operating profit, we have

$$\begin{aligned} PV_{\text{operating profit}} &= \pi_T \left(1 + 0.95^1 + 0.95^2 + 0.95^3 + 0.95^4\right) \\ &= 19{,}000{,}000 \times 4.52 = 85.88\text{M} \end{aligned}$$

and for the fixed cost

$$\begin{aligned} PV_{\text{fixed cost}} &= F\left(1 + 1.10^{-1} + 1.10^{-2} + 1.10^{-3} + 1.10^{-4}\right) \\ &= 15{,}000{,}000 \times 4.17 = 62.55\text{M} \end{aligned}$$

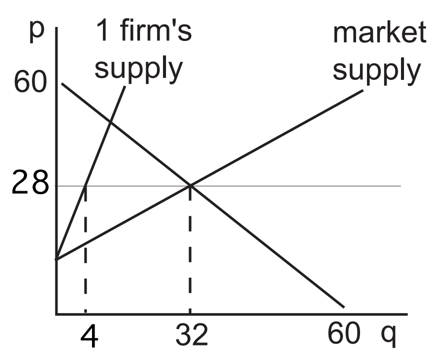
Thus, the total value of the factory is $85.88\text{M} - 62.55\text{M} = 23.33\text{M}$.

4. *EightFirms_a*. Each firm has marginal cost curve $MC(q) = 12 + 4q$. Since each firm will optimally set price equal to marginal cost, we invert this curve and each firm has supply curve $Q_{si}(p) = \frac{1}{4}p - 3$. Then market supply is eight times this, or $Q_s(p) = 2p - 24$.

   Market equilibrium occurs where supply equals demand,

   $$2p - 24 = 60 - p \Rightarrow 3p = 84 \Rightarrow p = 28$$

   Each firm produces $Q_{si}(28) = \frac{1}{4}28 - 3 = 4$.



5. *Long_a*. In the long run, there will be entry if $p > AC$ and exit if $p < AC$. Therefore we are looking for a point where both $p = MC$ (short-run optimizing) and $p = AC$ (long-run equilibrium). The only such point is where:

$$\begin{aligned}
MC(q) &= AC(q) \\
100q &= \frac{1000}{q} + 50q \\
50q &= \frac{1000}{q} \\
q^2 &= 20 \\
q &= 4.47 \\
p = MC(4.47) &= 447
\end{aligned}$$