ECON 110, Professor Hogendorn

Problem Set 5

1. *Water.* The government has offered to give you a monopoly if you will provide water to a city. The inverse demand curve is $p(Q) = 1000 - 0.01Q$ and the average cost curve is $AC(Q) = \frac{25{,}000{,}000}{Q} + 100$.

   (a) What are the marginal revenue and marginal cost curves?

   (b) What is the optimal price you should charge and quantity you should produce? What is the profit of the monopolist?

   (c) Graph this situation carefully.

2. *Campbell.* Campbell Soup works very hard to improve productivity at its factories.

   (a) Suppose that Campbell is able to change its production function from $f(L) = 75L^{1/2}$ to $f(L) = 85L^{1/2}$. What is its derived labor demand curve for both cases?

   (b) One big Campbell factory is in the town of Maxton, NC. Draw the labor market of Maxton. Justify the way you draw the labor supply curve and the labor demand curve for the town. (Note Campbell is not the only employer in Maxton.)

   (c) Suppose that Campbell is a perfect competitor in the soup market, with $p = 2.5$. Also suppose Campbell is a price-taker in the labor market, with $w = 10$. For the case of $f(L) = 85L^{1/2}$, what is Campbell's optimal level of employment, optimal output, and operating profit.

   (d) Suppose that Campbell actually has a near monopoly on the American soup market, with an inverse demand curve of $p(Q) =$

$5 - 0.011Q$. Continue to assume Campbell must pay $w = 10$ and that it has the production function $f(L) = 85L^{1/2}$ mentioned above. What is the monopoly profit maximizing quantity and price? (Note that you will have to rewrite the proftit function to reflect how price changes with quantity, which in turn changes with labor.)

3. *Revolution.* After Wesleyan, you take a job with McCoy Consulting. It was a tough decision because McCoy's big rival, Delight Consulting, was also recruiting you. And now the pressure is on because you are making a big presentation to Dolty, an auto parts manufacturer which is a perfectly competitive firm.

   (a) The perfectly competitive price of a car bumper is \$500. Dolty uses S tons of steel to make q units of bumpers according to the production function $q = f(S) = 1000S^{\frac{1}{12}}$. Steel is the only variable factor in this problem – you can think of it just like labor. The price of steel is \$800 per ton. What is Dolty's operating profit function $\pi(q)$?

   (b) Write down the first order condition for profit maximization and explain the economic logic behind it. You don't actually have to solve it.

   (c) After you have shown the above, a team from Delight bursts into the room. Their young leader, Trinity Amherst-Brown, says "Delight has a revolutionary new way to manage your firm. Don't think about bumpers, like these dinosaurs from McCoy! Instead, decide how much steel to buy!" She proceeds to write operating profit function $\pi(S)$. Assuming she does this correctly, what does she write down? Show the condition for profit maximization using this function.

   (d) Now it's up to you to save McCoy's reputation. Argue (in words) that the profit maximization condition for Ms. Amherst-Brown's

method is exactly the same as the profit maximization condition in your method, and that Delight has no revolutionary management technique.

Review Problems only, not to turn in:

4. *NaturalGas.* One of the most important energy trends of the past year has been the decline in natural gas prices in the United States. Measured per MMBtu (million British thermal units), they have declined from $4 to $2 from 2011 to 2012.

   (a) Suppose there is a perfectly competitive, profit-maximizing electricity generating firm that uses natural gas to produce electricity. Suppose the price of the electricity was $90 per MWh (megawatt-hour) in both 2011 and 2012. If the firm can freely adjust its input of natural gas, what was the marginal product of an MMBtu of natural gas in 2011 and in 2012?

   (b) Explain the logic behind why the firm let the marginal product fall in part (a).

   (c) Graph the firm's demand curve (the price), the average cost curve (assume there are some fixed costs), and the marginal cost curve. Draw it so the firm had price equal to average cost in 2011. Now show how the curves shift in 2012, and show whether the firm is making rents in 2012. (Think carefully how the AC curve shifts since the only change is in variable costs, not fixed costs.)

   (d) In the long run, if nothing else changes, what will happen? Illustrate with a graph of the overall electricity market.

5. *Generators.* It is 2 in the afternoon on a hot day in July. Everyone in the city has their air conditioning turned up, with the result that

the typical household demands 6 kWh (kilowatt hours) of electricity during the 2-3pm time slot. Their demand is perfectly inelastic because they are so hot, they don't care about the price!

There are two generating companies (GenCos) serving this city. Each one operates an oil-fired power plant that can produce electricity according to the production function

$$f(g) = (540g)^{\frac{1}{3}}$$

where g is gallons of oil and q is kWh of electricity per household in the city. The price of oil is 200¢ per gallon. Each GenCo also has a fixed cost of 20¢ per household. The price of electricity is p and the GenCos are perfect competitors (even though there's only two of them).

(a) The managers at GenCo A like to maximize their profits in terms of the quantity $q$of electricity per household that they produce. Write down the GenCo A profit function $\Pi(q)$. Derive GenCo A's supply curve.

(b) The managers at GenCo B work differently. They figure out how much oil to buy to maximize profits. Write down the GenCo B profit function $\Pi(g)$. Derive GenCo B's supply curve (this will require an extra step relative to your answer for (a)).

(c) Describe in words why the two GenCos end up with the same supply curves.

(d) What is the market equilibrium price of electricity? Draw a graph of the markeget equilibrium.

(e) Derive and graph the marginal and average cost curves for one of the firms. At the market equilibrium price, calculate and label on the graph the profit or loss of the firm.

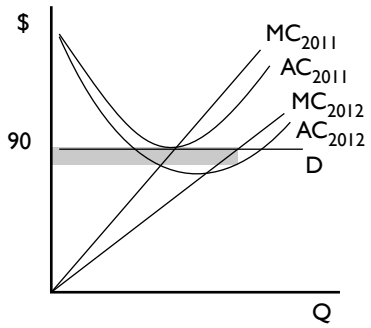
Answers to Review Problems:

4. *NaturalGas_a.*

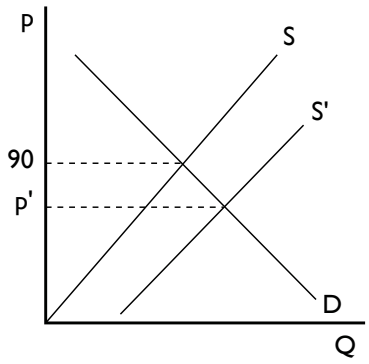
   (a) A perfectly competitive, profit maximizing firm always sets price of output times marginal product of a factor equal to the price of the factor. For example, for labor it sets $pMP_L = w$. For gas the equation is $90MP_g = p_g$. In 2011, this gives $MP_g = 4/90$ and in 2012 it gives $MP_g = 2/90$.

   (b) Since the firm is profit maximizing, it keeps on using more gas until the value of the product of the marginal unit of gas equals the cost the the marginal unit of gas. With the gas price lower, the firm makes higher profits by using more gas even though diminishing returns set in and the marginal product of gas is lower.

   (c) Since the price of a factor of production has fallen, both marginal and average cost curves shift down. The AC curve shifts down less at low quantities and more at high quantities since the importance of variable costs becomes greater at larger quantities. Since the firm made zero profit before, it must now be making rents since its costs are lower. The rents are shown by the shaded box.



   (d) In the long run, the rents will attract more firms to enter the electricity market by using natural gas. The increased supply will lower electricity prices, eventually pushing them down until a typical natural gas generator again earns zero profits.



5. *Generators_a.*

   (a) GenCo A's conditional factor demand for oil is found by inverting the production function:

   $$540g = q^3 \Rightarrow g(q) = \frac{q^3}{540}$$

   Profits are revenue minus fixed cost minus variable cost:

   $$\Pi(q) = pq - TC(q) = pq - 20 - 200\frac{q^3}{540}$$

   To maximize profits, take the derivative and set equal to 0. This is equivalent to the price-equals-marginal-cost condition.

   $$\mathrm{p} - \mathrm{MC(q)} = \mathrm{p} - 600\frac{\mathrm{q}^2}{540} = 0$$

   Finally, the supply curve is quantity as a function of price:

   $$\mathrm{q}^2 = \frac{9}{10}\mathrm{p} \Rightarrow \mathrm{s(p)} = \left(\frac{9}{10}\mathrm{p}\right)^{1/2}$$

   (b) With this method, costs are easy, just $20 + 200g$. Revenue is equal to price times the amount of production:

   $$\Pi(g) = pq(g) - 20 - 200g = p(540g)^{\frac{1}{3}} - 200g$$

To maximize profits, take the derivative and set equal to 0. This is equivalent to the price-times-marginal-product-equals-factor-price condition.

$$pMP_g - 200 = p\frac{1}{3}540^{\frac{1}{3}}g^{-\frac{2}{3}} - 200 = 0$$

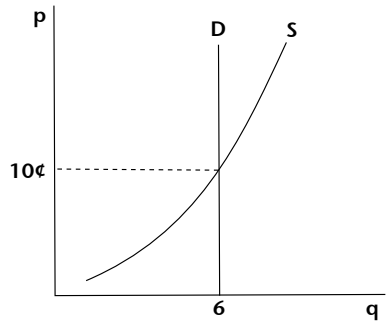
Now finding the supply curve takes two steps. First, find the unconditional factor demand $g(p)$, then use the production function to turn this into quantity produced as a function of price:

$$g^{-\frac{2}{3}} = \frac{600}{p540^{1/3}}$$
$$g(p) = \frac{600^{-3/2}}{p^{-3/2}540^{-1/2}}$$
$$s(p) = q(g(p)) = 540^{1/3}\frac{600^{-1/2}}{p^{-1/2}540^{-1/6}}$$
$$s(p) = 540^{1/2}\frac{600^{-1/2}}{p^{-1/2}540^{-1/6}}$$
$$s(p) = 540^{1/2}p^{1/2}600^{-1/2}$$

(c) Both gencos make their decisions on the basis of "should the company do a little more." GenCo A's condition says that additional electricity should be produced until the marginal cost of another unit equals the revenue from selling it. GenCo B's condition says that additional oil should be purchased until the cost of the oil equals the revenue generated from the marginal product (measured in electricity) made from the oil. These conditions are restatements of the same idea; both say "is it profitable to do a little more of this activity."

(d) Market equilibrium occurs when supply of electricity equals demand for electricity. Demand is just 6. Market supply is the sum of the supply curves of GenCos A and B. Thus,

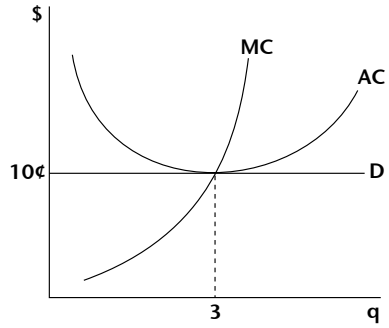
$$2s(p) = 2 \cdot 540^{1/2}p^{1/2}600^{-1/2} = 6$$
$$0.95p^{1/2} = 3$$
$$p = 10\text{ć}$$



(e) We already found TC above, so applying the definitions of AC and MC gives:

$$TC(q) = 20 + 200\frac{q^3}{540}$$
$$AC(q) = \frac{TC(q)}{q} = \frac{20}{q} + 200\frac{q^2}{540}$$
$$MC(q) = \frac{dTC(q)}{dq} = \frac{600}{540}q^2$$

To draw the graph, we need to find that $AC(3) = 10$.



Since the average cost is equal to price, profit is:

$$\Pi(3) = (p - AC(3))3 = (10 - 10)3 = 0$$

Thus, there is no super-normal profit or loss.