ECON 110, Professor Hogendorn

Problem Set 5 Answers

1. *Water_a.*

   (a) Total revenue and marginal revenue are:

   $$\begin{aligned} TR(q) &= p(q) \times q = (1000 - 0.01q)q \\ MR(q) &= \frac{dTR}{dQ} = 1000 - 0.01q - 0.01q = 1000 - 0.02q \end{aligned}$$

   Total cost and marginal cost are:

   $$\begin{aligned} TC(q) &= 25,000,000 + 100q \\ MC(q) &= 100 \end{aligned}$$

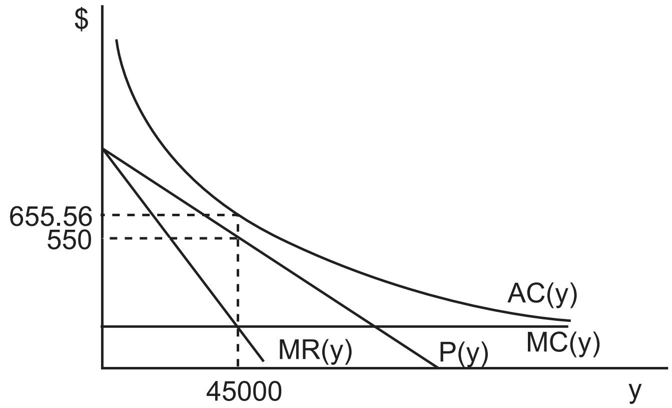
   (b) Profit-maximizing behavior is to set marginal revenue equal to marginal cost:

   $$1000 - 0.02q = 100 \Rightarrow q = 45000$$

   The price at this output is $p(45,000) = 1000 - 0.01 \cdot 45,000 = 550$. The profit is:

   $$\begin{aligned} \Pi(45,000) &= (p(45,000) - AC(45,000))45,000 \\ &= (550 - 655.56)45,000 \\ &= -4,750,000 \end{aligned}$$

   (c) The key here is that the AC curve lies everywhere above the demand curve, so there's no way the monopoly can avoid a loss, even at the maximum "profit" level.



2. *Campbell_a.*

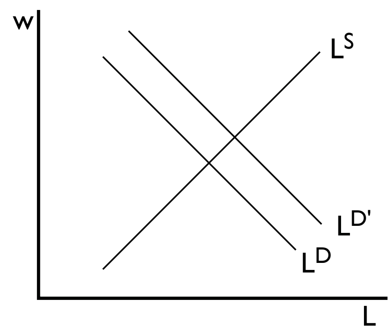
(a) We have derived that the profit maximizing condition for a firm can be expressed as $pMP_L = w$. For these particular production functions this gives

$$p37.5L^{-1/2} = w \Rightarrow L^{1/2} = \frac{37.5p}{w} \Rightarrow L(w,p) = \frac{1406.25p^2}{w^2}$$

$$p42.5L^{-1/2} = w \Rightarrow L^{1/2} = \frac{42.5p}{w} \Rightarrow L(w,p) = \frac{1806.25p^2}{w^2}$$

(b) Labor demand will be downward-sloping because at lower wages, firms are willing to let the marginal product of labor fall. This is true of Campbell and all the other employers. Assuming Cambell is large enough, the shift shown in part (b) may be apparent in the market demand curve as well.

Labor supply is likely to be upward-sloping. Although most people need to work no matter what, they don't need to work in Maxton, and people from out of town can work there.



(c) In part (a), we found that $pMP_L = w$ for this production function implies a labor demand curve $L(w,p)$. If we plug $p = 2.5$ and $w = 10$ into this demand curve we find that Campbell hires

$$L(10, 2.5) = \frac{1806.25 \cdot 2.5^2}{10^2} = 113$$

units of labor. Based on the production function, they produce $f(113) = 85 \cdot 113^{1/2} = 903$ cans of soup. Their revenue is $2.5 \cdot 903 = 2257$ and their total variable cost from wages is $wL = 10 \cdot 113 = 1130$ so their operating profit is $2257 - 1130 = 1127$.

(d) The total revenue of Campbell is

$$\begin{aligned} TR = p(Q)Q &= (5 - 0.011Q)Q \\ &= (5 - 0.011 \cdot 85L^{1/2})85L^{1/2} \\ &= 425L^{1/2} - 79.5L \end{aligned}$$

so the marginal revenue product of labor is $MRP = \frac{dTR}{dL} = 212.5L^{-1/2} - 79.5$.

Then by setting marginal revenue product of labor equal to the wage we find that

$$\begin{aligned} 212.5L^{-1/2} - 79.5 &= 10 \\ L^{-1/2} &= .42 \\ L_M &= 5.7 \end{aligned}$$

and plugging into the production function we get $f(5.7) = 85 \cdot 5.7^{1/2} = 203$ cans of soup. They'll sell these at a price of $P(203) = 5 - 0.011 \cdot 203 = 2.767$ each.

3. *Revolution_a.*

   (a) The profit function is total revenue minus total cost. To find the total cost, we need to know how much steel is used per bumper, which is just the inverse of the production function: $S(q) = (0.001q)^{12}$. Then:

   $$\Pi(q) = 500q - 800(0.001q)^{12}$$

   (b) To maximize the profit function, take the derivative and set equal to 0:

   $$\frac{d\Pi}{dq} = 500 - 12 \cdot 800(0.001q)^{11} \cdot 0.001 = 0$$

   In words, the condition is price (or marginal revenue) equals marginal cost. The logic is that the firm should keep producing bumpers as long as the marginal cost of a bumper is less than the price the firm receives for it. As more bumpers are produced, the marginal cost rises, and once it is equal to the price, no further profits can be made by producing more bumpers. In fact, more bumpers would reduce profits.

   (c) TA-B's method also starts with profits equal total revenue minus total cost, but they are all measured in terms of S. Since the amount of bumpers produced is always $q = 1000S^{\frac{1}{12}}$, TA-B's profit function is:

   $$\Pi(S) = 500 \cdot 1000S^{\frac{1}{12}} - 800S$$

   She also takes the derivative and sets equal to 0:

   $$\frac{d\Pi}{dS} = \frac{1}{12}500 \cdot 1000S^{\frac{-11}{12}} - 800 = 0$$

(d) TA-B's s method is identical. Because the production function provides a direct relationship between S and y, either decision making process yields the same result. McCoy's condition says that additional output should be produced until the cost of another unit equals the revenue from selling it. Delight's condition says that additional steel should be purchased until the cost of the steel equals the revenue generated from selling bumpers made from the steel. These conditions are restatements of the same idea.