ECON 110, Professor Hogendorn

Problem Set 6

1. You win a $10 million lottery jackpot, but then you find that the payout is actually $2 million per year for 5 years. Assuming there is no inflation over that time, and assuming a discount rate of $r = 0.05$, what is the present value of your jackpot?

2. *SmallCountry.* Remember that a country's supply of loanable funds is the *net* supply after households that borrow are subtracted from those who save. Suppose there is a small country with 1000 households. 700 of these have a savings function $S_H(r) = 50r$, where r is the real rate of return on capital. The remaining 300 households have savings function $S_L(r) = -1 + 10r$. (You can imagine that the number of households and the amount of savings are in thousands.)

   (a) Graph the individual and aggregate savings functions. Describe in words what happens to both types of household and the whole country when the real interest rate rises from 3% to 11%.

   (b) There are 100 firms, and each firm has a firm-level investment demand function $D_K(r) = 10/r$. Find and graph the aggregate investment function for the whole country.

   (c) Show that the equilibrium interest rate in this country is 16.6% (rounded to one decimal).

3. *Enough.* Suppose you have a two-period budgeting problem, with dollars today and dollars in the future. Your endowment in the first period (the first half of your life) is $M_t = \$1$, which we interpret to

mean your income will be 1 million dollars in the first half of your life. In the second half of your life, you will receive a higher income $M_f = \$4$. There is no inflation in this problem.

(a) Draw your endowment and a budget line where the real interest rate is $r = 0.08$. (Sorry, you're too risky to be able to borrow at $r = 0.05$.)

(b) If you just wanted to spend it all right now, what is the present value of your entire endowment?

(c) Do you have enough endowment to borrow money and consume $C_t = \$2.5$ in the first half of your life and $C_f = \$2.5$ in the second half of your life? You'll have to borrow \$1.5 in the present, so the question is how much is leftover in the future after you pay back the \$1.5 with interest. You can show this on your diagram by plotting the (2.5,2.5) point and seeing if it is above or below the budget line.

(d) How would the interest rate have to change to put the (2.5,2.5) point *on* your budget line? What is the intution for why the interest rate would have to go up/down to make the point lie along the budget line?

(e) Suppose you decided instead to save \$0.2 in the first half of your life. How much could you consume in the second half? Show on the budget line diagram.

Review Problems only, not to turn in:

4. *UncleKarl.* Your Uncle Karl gives you 20,000 dollars of capital. You can invest some of the capital in an Internet business venture that depends on people watching your videos.

(a) For each dollar of capital invested over the course of one year, do you think it is more reasonable to let your real cost of that

capital be \$0.05, \$0.10, or \$0.15? Discuss with regard to the riskinesss of this business.

(b) To simplify, assume no labor is involved in this business; the only factor is capital. Your production function is $f(K) = 10K^{9/10}$, where output is measured in the number of videos. You must also use \$5,000 more of capital to pay a fixed cost to get started. What are the equations for your total, average, and marginal cost curves, using your answer to (a)? Graph the AC and MC curves.

(c) If each video brings you revenue of \$0.04, how much capital should you invest in this business? Show this on your graph. Do you earn a competitive rate of return on your capital, or do you receive rents?

Answers to Review Problems:

4. *UncleKarl_a.*

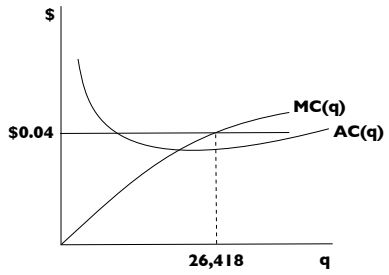
(a) We know that over the long run, low-risk investments seem to return about 5%. Since this is a risky business, a 5-cent cost of capital seems too low. If it were a larger more mature business, 10% might be a good discount rate to use, but this is a tiny operation. So we're probably talking a much higher level of risk and a cost of capital of \$0.15 would be more appropriate. (Indeed, a cost of capital of more like \$0.40 might be reasonable.)

(b) Since $f(K) = 10K^{9/10}$, you need $K(q) = \frac{q}{10}^{10/9}$ units of capital to produce output q. Since capital costs \$0.15, and you have an additional fixed cost of \$5000 that also comes out of

capital, the cost curves are:

$$TC(q) = 0.15\left(5000 + \frac{q}{10}^{10/9}\right) = 750 + 0.0116q^{10/9}$$
$$AC(q) = \frac{TC(q)}{q} = \frac{750}{q} + 0.0116q^{1/9}$$
$$MC(q) = \frac{dTC(q)}{dq} = 0.0129q^{1/9}$$

If you draw the graph exactly, it is a little strange because marginal cost is concave:



(c) Your profit maximizing quantity is where marginal revenue product of capital equals the cost of capital:

$$MC(q) = p \Rightarrow 0.0129q^{1/9} = 0.04 \Rightarrow q^* = 26,418$$

At that quantity, you need to invest $K(26,418) = 6,340$ dollars of capital plus the 5,000 dollar startup cost. Given that your cost of capital is \$0.15, your total costs are \$1,701. Your total revenue is $q^* = 0.04 \times 26,418 = \$1,056.72$. Thus you actually lose money on this investment, since your revenues are lower than your costs, including the proper cost of capital.