

ECON 110, Professor Hogendorn

Problem Set 8

1. *OldGermans*. In Germany, the birth rate is low and the population is ageing. As a result, the working age population is falling at about 0.2% per year. It has been suggested that this population decline puts the German economy at risk. This question asks you to use our simple neoclassical model to evaluate that claim.

Let there be $\mathcal{L} = 243$ German workers who inelastically supply labor and who spend all of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. (Aggregate meaning we treat all the firms as if there were just 1.) There is no money, so the price of one beer is one beer.

- (a) Find the equilibrium real wage in the labor market and graph the labor market.
 - (b) Verify that there is also equilibrium in the beer market and graph the production function. What share of workers' income comes from wages and what share from dividends?
 - (c) Suppose that over 10 years, the German population falls and there are only $\mathcal{L}' = 198$ workers. Find the new general equilibrium.
2. *Botswana*. In his book *Globalization and its Discontents*, pg. 38, Joseph Stiglitz criticized the IMF's policy toward Botswana in 1981. He uses this as one example of a larger critique of the "Washington Consensus" policy toward developing nations. This problem uses approximately accurate data to analyze the situation.

First, let's normalize Botswana's working population to 100, and let labor supply be $L^s = 100$. Let Botswana have an economy-wide production function

$$Y = f(L) = 117.5L^{1/2}$$

and assume the firms represented by this function are owned by the workers. Set the price of Y equal to 1, and note that with the given production function, GDP at full employment is 1,175 million US dollars.

- (a) Find the equilibrium real wage in Botswana's labor market and graph the labor market. Graph the production function.
- (b) Now let's examine the capital market in 1980. Private Botswanans were saving 5% of GDP (assume perfectly inelastic with respect to the real interest rate). The government was spending 33% of GDP and collecting taxes of 34% of GDP. Firms' investment demand function was $I(r) = 728 - 3520r$. Graph the domestic capital market and show the equilibrium real interest rate.
- (c) Actually, Botswana's real interest rate in 1980 was 10%, lower than in part (b). This was because Botswana could borrow in the world capital market. Redraw your capital market graph to show this lower real interest rate. How much investment took place in Botswana? What were foreign capital inflows?
- (d) Recall that total income from wages plus dividends has to equal consumption plus savings plus taxes. Find this for Botswana in 1980. Then recall that total output has to equal consumption plus investment plus government plus net exports. Find this for Botswana in 1980. Note: you just found consumption; investment and government were given in part (b) and (c); net exports is the residual that makes total income equal to total output.

- (e) Botswana faced two negative shocks in 1981 due to drought and problems in the diamond industry. We'll model this by saying that the production function changed for the worse to $Y = f(L) = 103.8L^{1/2}$. Show the new real wage and the new real GDP. The government of Botswana continued to collect taxes of 34% of Y' . The IMF advised Botswana to cut government spending, but instead it spent 36% of Y' . Private saving was 5% of Y' and investment was the same as above. Graph what happened in the capital market assuming the world real interest rate remained at 10%.
3. *RateSpread*. Consider a small country in the global economy. There is a world real interest rate of 2%, i.e. $r = 0.02$. This country has perfectly inelastic domestic savings of 100 dollars. The demand for investment in this country is $I = 150 - 1000r$.
- (a) Draw the capital market diagram for this country. Remember, it doesn't clear because the country can borrow and lend in international markets. Label the amount invested and the amount of net capital flows (NCF) into or out of the country.
- (b) Bad news. This country has become unstable and is now perceived as very risky. As a result, a "spread" of 2% opens up in the capital market. This means that now the country can borrow or lend internationally at $r = 0.04$. Show on the graph.
- (c) Suppose that inflation in this country has been consistently equal to 3%. Suppose there were a bond issued before the instability began. It has a coupon rate of 5%. What is the most reasonable guess for its yield now: 3%, 5%, or 7%? Explain, including whether the price of this bond rises, falls, or stays the same.

Review Problems only, not to turn in:

4. *GrowingChina*. This problem discusses the Malthusian trap that has worried China for centuries and that the country has now escaped. Let there be $\mathcal{L} = 1000$ Chinese workers who inelastically supply labor and who spend all of their income on rice. These workers own the Chinese rice firms which have aggregate production function $Y = f(L, K) = A(hL)^{2/3}K^{1/3}$. (Aggregate meaning we treat all the firms as if there were just 1.) Let $A = 3.33$, $h = 1$, $p = 1$ and let $K = 729$. Note that the Chinese capital stock is constant until part (d) of this problem.
- (a) Find the equilibrium real wage and graph the labor market.
 - (b) Verify that there is also equilibrium in the rice market and graph the production function. What is output per worker (Y/\mathcal{L})?
 - (c) Suppose that over several years, the Chinese workforce rises to 1,728 workers. If nothing else changes, what is the new general equilibrium (the new wage and the new output per worker)? Why don't these new workers produce enough to keep the output per worker at least as high as before?
 - (d) Consider the following changes to the production function: an increase in A , an increase in K , and an increase h . How would each of these help China escape the Malthusian trap? What is the name for each of these sources of growth?
5. *Uchitelle*. The following are quotes from an op-ed by Louis Uchitelle that appeared in the New York Times on August 25, 2002. It provides some food for thought, but we can evaluate the arguments a lot more clearly in a simple macroeconomic model. At the end of the article is a suggestion for trying to model Uchitelle's idea.

In Alice in Wonderland fashion, we talk of expansion and ignore the contraction all around us. We convince ourselves that out of cost-cutting will come prosperity. But while cost-cutting can lift a single company or two, when practiced widely enough it can pull down an economy. And that is happening today.

... consider what happens in an imaginary country where Burger King and McDonald's are the entire business sector and the total national output 100 hamburgers a day, evenly divided between the companies matches the demand from this nation's consumers. Demand and sales revenue, however, stay flat. So Burger King lays off two workers and uses the saved wages partly to fatten profits and partly to discount prices by just enough to take sales and revenue away from McDonald's. And McDonald's responds in kind. But soon, the four laid-off workers, with little income, buy fewer hamburgers, and the nation's total consumption drops to 95 hamburgers a day. That sets off another round of cost-cutting and price discounting, and our imaginary nation sinks gradually into stagnation or deep recession not unlike America in the 1930's.

Let the entire population of the economy be 32 workers who inelastically supply labor and who spend all of their income on hamburgers. Let McDonald's and Burger King be identical firms that each have production function $f(L) = 25L^{0.25}$. Let them both behave as perfect competitors. Let the price of one burger be one burger.

- (a) Find the equilibrium real wage in the labor market. Remember that there are TWO firms, so the total demand for labor

is the sum of each firm's demand for labor. Illustrate with a graph.

- (b) Verify that there is also equilibrium in the hamburger market and comment on the sources of the workers' total income.
- (c) Suppose that the two firms each laid off 2 workers as Uchitelle wrote. Assume the laid-off workers get no income whatsoever. Also suppose that the remaining 28 workers receive the same wage as before. Show the situation on a labor market diagram. Are the firms' profits higher? What about the workers' incomes?

6. *OldGermansSave*. As in *OldGermans*, there are 243 German workers who inelastically supply labor, but now they save 100 beers (in total) for the future and spend the rest of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. The German beer firms have aggregate investment demand of $I = 1200/r$, where r is the real interest rate.

- (a) Find the equilibrium real wage in the labor market and graph the labor market. Verify that there is also equilibrium in the beer market and graph the production function. (This just repeats *OldGermans* parts (a) and (b)).
- (b) Graph the capital market. What is the equilibrium real interest rate?
- (c) What happens if Germans become more pessimistic and start saving 110 beers?
- (d) Go back to just 100 beers saved. What happens if the German government levies taxes of 60 beers but German Chancellor Angela Merkel drinks 80 beers?

7. *Fear-goods*. This problem shows how in the neoclassical long-run macro model, widespread fear across an economy will not cause a recession! This is an important and comforting insight for the long run, but on the other hand, in the long run we are all dead...

Suppose the production function for the one representative firm in the economy is $Y = f(L) = 20L^{4/5}$. There are $\mathcal{L} = 40$ workers who inelastically supply labor.

- (a) Show that the labor demand curve is $L(w) = (16/w)^5$, graph the labor market, and show the equilibrium real wage.
- (b) Verify the national income accounts identity, i.e. that income from wages and dividends (which equals consumption) equals output (all of which is also consumption).
- (c) Now suppose that people in this country hear about the financial crisis. Everyone becomes very fearful of the future. The firm shifts down its labor demand curve to $L(w) = (8/w)^5$ – even though this is not profit maximizing because the production function remains unchanged. Assuming the labor market still clears, what happens to the wage, income from wages, income from dividends, and output?

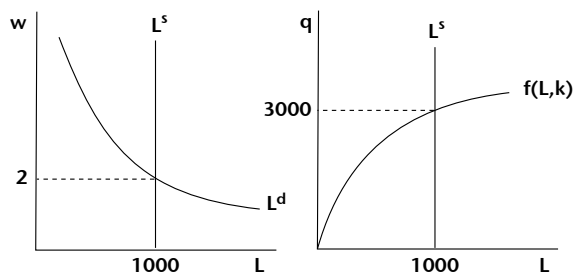
Answers to Review Problems:

4. *GrowingChina_a*.

- (a) We can find labor demand using $pMP_L = w$, so,

$$\begin{aligned} \frac{2}{3} \cdot Ah^{2/3}L^{-1/3}K^{1/3} &= w \\ \frac{2}{3} \cdot 3.33L^{-1/3}729^{1/3} &= w \\ 20L^{-1/3} = w &\Rightarrow L^d(w) = \left(\frac{20}{w}\right)^3 \end{aligned}$$

Setting $L^d = L^s = 1000$ gives equilibrium real wage $w = 2$.



- (b) The total costs of the firm are $wL = 2 \cdot 1000 = 2000$. The total revenues are $py = 1 \cdot f(1000, 729) = 3000$. Thus the profits, paid as dividends, are 1000. The firm's output is 3000.

Workers earn total wages of $wL = 2000$ and total dividends of 1000. Their total consumption of rice is thus 3000, so there is equilibrium. Output per worker is 3.

- (c) L^d is the same as before, but now setting $L^d = L^s = 1728$, gives an equilibrium real wage of $w = 1.67$. Firm output is $f(1728) = 4326$, and output per worker is 2.5.

The output per worker falls because the marginal product of labor falls. Even though the additional workers produce (and consume) more, their addition on the margin is less than before. The economy is running into decreasing returns to labor and thus into a possible Malthusian trap.

- (d) An increase in A is an increase in total factor productivity, sometimes simply called technology. It would increase the marginal products of both labor and capital, thus raising the wage and output per worker.

An increase in K is called capital deepening. This raises the marginal product of labor (though it lowers the marginal product of capital) and thus raises wages. Each worker has more capital to work with.

An increase in h is called human capital formation. Although it does not change the degree of diminishing returns to *skill-*

adjusted hours of labor, it does increase the marginal product of each generic hour of labor.

5. *Uchitelle_a.*

- (a) The profit of one of the firms is $\pi(L) = p \cdot 25L^{1/4} - wL$. The first order condition for the optimal L to demand is

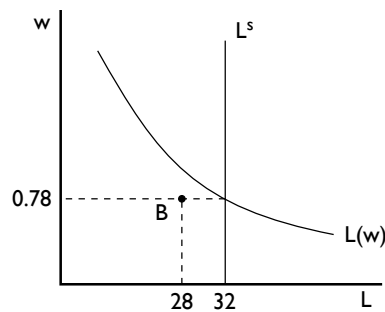
$$\frac{d\pi}{dL} = 6.25L^{-3/4} - w = 0$$

Solving for L , we find that the firm's labor demand is

$$L^D = 11.5w^{-4/3}$$

Setting labor supply equal to *market* labor demand gives us:

$$32 = 23w^{-4/3} \Rightarrow w^* = 0.78$$



- (b) At this wage, each firm hires $L^D = 16$ workers and produces an output of $f(16) = 50$ hamburgers. Each firm makes a profit of $\pi(16) = 50 - 0.78 \cdot 16 = 37.52$. The income of the consumers is the total wage bill of $0.78 \cdot 32 = 25$ plus the dividends earned from owning the firms, for a total of $25 + 2 \cdot 37.5 = 100$. With nothing else to buy, this means consumers demand 100 hamburgers, which is the total output of the firms.
- (c) Each firm now hires 14 workers, although this is not on their correctly-calculated labor demand curve as shown by point

B in the diagram. Each firm's output is now $f(14) = 48.36$ hamburgers. The wage bill is only $0.78 \cdot 14 = 10.92$, so the profits of a firm are $\pi(14) = 48.36 - 10.92 = 37.44$. So firm profits fall slightly, which makes sense since they are no longer profit-maximizing. This implies that the dividend portion of household income also falls slightly.

The wage income portion of household income clearly falls, since fewer people are employed, although this reduction falls entirely on the 4 unemployed workers.

6. *OldGermansSave_a.*

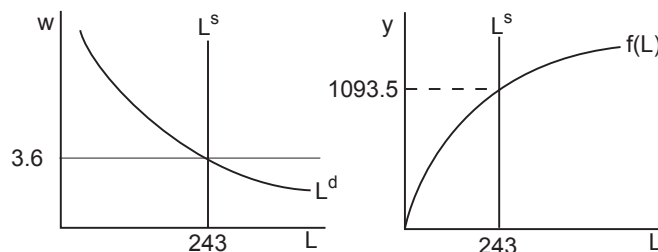
(a) We can find labor demand using $pMP_L = w$, so,

$$1 \cdot \frac{4 \cdot 54}{5 \cdot 4} L^{-1/5} = w \Rightarrow L^d = \left(\frac{54}{5w} \right)^5$$

Setting $L^d = L^s = 243$ gives an equilibrium real wage of $w = 3.6$.

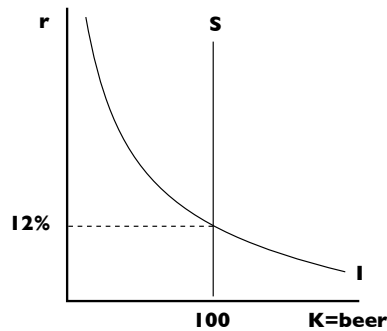
The total costs of the firm are $wL = 3.6 \cdot 243 = 874.8$. The total revenues are $py = 1 \cdot f(243) = 1093.5$. Thus the profits, paid as dividends, are 218.7. The firm's output is 1093.5.

Workers earn total wages of $wL = 874.8$ and total dividends of 218.7. Their total consumption of beer is thus $1093.5 - 100 = 993.5$, and the remaining 100 beers are saved, so there is equilibrium.



(b) The equilibrium real interest rate is found by setting

$$I = S \Rightarrow \frac{1200}{r} = 100 \Rightarrow r = 12\%$$



(c) The higher savings reduces the real interest rate:

$$I = S \Rightarrow \frac{1200}{r} = 110 \Rightarrow r = 10.9\%$$

(d) Consumption of beers falls to $1093.5 - 100 - 60 = 933.5$. Private savings stays the same at 100. Government spending is 80, so $T - G = -20$, i.e. the government runs a deficit. National saving is then $100 - 20 = 80$. The real interest rate rises to

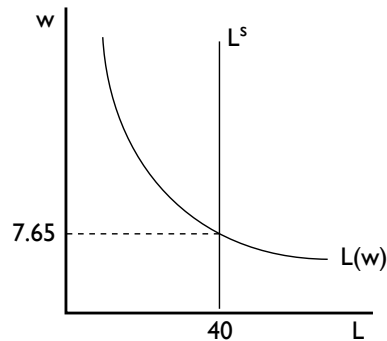
$$I = S \Rightarrow \frac{1200}{r} = 80 \Rightarrow r = 15\%$$

Note that the government deficit fully crowds out private investment, which falls from 100 to 80.

7. Fear-goods_a.

(a) Firms maximize profits by setting the marginal product of labor equal to the wage:

$$f'(L) = w \Rightarrow \frac{4}{5}20L^{-1/5} = w \Rightarrow L^{-1/5} = \frac{w}{16} \Rightarrow L(w) = \left(\frac{16}{w}\right)^5$$



- (b) Since the labor market clears, employment is 40, and output is $Y = f(40) = 382.5$.

Income is equal to wages plus dividends. Wages are $wL = 7.65 \cdot 40 = 306$. To find dividends, we need to find the profits of the firm:

$$\Pi = pq - wL = 1 \cdot 382.5 - 306 = 76.5$$

So total income is $306 + 76.5 = 382.5$ which does indeed equal output.

- (c) The labor market now equilibrates off the new, irrational labor demand curve, so

$$\left(\frac{8}{w}\right)^5 = 40 \Rightarrow w = 3.83$$

Not surprising, workers' wages fall because the firms' collective fear has essentially the same effect as if they all colluded to reduce wages. But since this is a neoclassical model, the labor market does still clear, and all 40 workers are still employed. That means that output is still $Y = f(40) = 382.5$.

It remains to be seen if there is really income to pay for this output. Total wages are now only $3.83 \cdot 40 = 153.2$. But firm profits now rise (due to the lower labor costs) to $1 \cdot 382.5 - 153.2 = 229.3$. Therefore, dividends go up a lot, and total income is still $153.2 + 229.3 = 382.5$, exactly enough to equal output.