

## Second Midterm Exam

Each part of a question (a, b, c, etc.) is worth 5 points. Make sure to allot your time accordingly. Total of 35 points, -1 for messiness, -2 for extreme messiness.

If possible, please write on one side of the blue book pages. When you are finished, please keep the exam sheet and hand in your blue book. Thanks.

1. *12 firms* There is a firm with production function

$$q = f(L, K) = L^{1/2} + K^{1/2}$$

This firm is initially stuck in the short run with  $\bar{K} = 16$  which cannot be changed. The wage is  $w = 3$  and the price of capital is  $r = 4$ .

- (a) Find the short run marginal cost curve and the short-run supply curve.
- (b) If there are 12 firms, and if market demand is  $q(p) = 96 - p$ , what is the short-run market equilibrium price?
- (c) What is the short-run average total cost? Is this firm making a loss, breaking even, or making a super-normal profit? Illustrate on a two-panel graph, one panel showing the market, the other showing the cost curves of an individual firm.

2. *Tax-rates.* As you know, the question of whether tax rates should be higher on richer versus poorer people is extremely contentious. This problem looks at the basis for a progressive tax, i.e. one where the rate is higher on richer people.

There are two people, both of whom have the same utility function:

$$u(x_i, G) = x_i^{1/2} + G^{1/2}$$

The variable  $x_i$  is the amount of numeraire consumed by person  $i$  and  $G = g_1 + g_2$  is a public good. The budget constraints for the two people are  $x_1 + g_1 = m_1$  and  $x_2 + g_2 = m_2$ . Note this means that the price of each good is just 1.

- (a) Write down the private utility maximization problem of person 1. (Person 1 takes the contribution of person 2,  $g_2$ , as given.) Solve the private problem using either the Lagrangian or the MRS method, your choice. Your solution should be a private demand curve  $g_1(m_1, g_2)$ .
- (b) Write the social planner's utility maximization problem, and explain in words how it differs from the private problem.
- (c) The planner's problem is easiest to solve by substitution. Note that the budget constraints imply that  $x_1 = m_1 - g_1$  and  $x_2 = m_2 - g_2$ . Making those substitutions, the planner's problem becomes a two-variable, unconstrained optimization problem. Solve it out to find the optimal levels of  $g_1$  and  $g_2$ .
- (d) Suppose that  $m_1 = 10$  and  $m_2 = 1$ . The tax rates in this problem are just  $g_1/m_1$  and  $g_2/m_2$ . Who pays a higher tax rate, the richer person or the poorer person? Explain in words why this is socially optimal.