

ECON 301, Professor Hogendorn

Review Problem Set 10

1. *Glendivot*. The Glendivot distillery makes Scotch whisky and then stores it for a period of time before selling it. The market price of a barrel of Glendivot is

$$V(t) = 480t - 12t^2$$

where t is the number of years of aging.

- (a) The distillery produces a private reserve which is consumed only by the owner's family. The objective is to produce the highest possible value Scotch without reference to costs. How long is the private reserve aged?
 - (b) The distillery also produces its regular product for commercial sale. It faces an interest rate of 5%, compounded continuously. How many years does it age its regular product?
 - (c) Suppose the market for Scotch is perfectly competitive and in long run equilibrium with no entry or exit of firms (a dubious assumption, but let's go with it). Assume there is only one cost the distillery incurs: the cost of distilling a barrel's worth in the first place. How much does it cost to distill a barrel? (Note: ignore the private reserve; that is private consumption by the owner.)
2. *Mackenzie*. Mackenzie Rembrandt had a famous ancestor who painted a family portrait that has been handed down to her. The value of this painting increases over time according to the function

$$V(t) = 10t - 2t^2$$

where t is the number of years starting now and V is measured in millions of dollars.

- (a) If Mackenzie's objective is to let the painting appreciate to its maximum value and then give it to a museum, when should she give away the painting?
- (b) Suppose instead that Mackenzie views the painting as an investment, and she can get a (continuously compounded) interest rate of 7%. When should she sell the painting?

Answers:

5. *Glendivot_a.*

- (a) To solve $\max_t V(t) = 480t - 12t^2$ find the FOC:

$$V'(t) = 480 - 24t = 0 \Rightarrow t = \frac{480}{24} = 20$$

- (b) To solve $\max_t PV(V(t)) = e^{-.05t}(480t - 12t^2)$ the FOC is:

$$\begin{aligned} \frac{dPV(V(t))}{dt} &= (480 - 24t)e^{-.05t} - .05e^{-.05t}(480t - 12t^2) = 0 \\ 480 - 24t - 24t + 0.6t^2 &= 0 \\ 0.6t^2 - 48t + 480 &= 0 \end{aligned}$$

Use the quadratic formula or a calculator to find that $t = 11.7$.

- (c) Using the best "technology" available (i.e. $t = 11.7$), there must be a present value of zero when discounted at the proper rate:

$$\begin{aligned} PV &= -c_B + e^{-.05 \cdot 11.7}(480 \cdot 11.7 - 12(11.7)^2) = 0 \\ c_B &= \$2,213.56 \end{aligned}$$

6. Mackenzie_a.

- (a) Just maximizing the value of the painting leads to the first order condition:

$$\frac{dV}{dt} = 10 - 4t = 0 \Rightarrow t = 2.5 \text{ years}$$

- (b) In this case, the problem is to maximize $\frac{V(t)}{e^{0.07t}}$, which leads to the first order condition:

$$(10 - 4t)e^{-0.07t} + (10t - 2t^2)(-0.07)e^{-0.07t} = 0$$

The exponential terms cancel, and we can simplify to:

$$0.14t^2 - 4.7t + 10 = 0$$

Using a calculator (or the quadratic formula), we find

$$t = 2.28 \text{ or } 31.29$$

Only the first answer is economically meaningful in this situation.