

ECON 301, Professor Hogendorn

Problem Set 5

1. *Educated Mothers*. A strong finding by development economists around the world is that when women are better educated, not only does their own standard of living rise, but so does their children's. The effect is largely due to the education of the mother herself, but also the average educational level of women in the community makes a difference. Suppose for example that a typical woman's utility function is $u_w(x, e) = x^{3/4}e^{1/4}$ where e is her own educational level and that a typical child's utility function is $u_c(e, E) = 12e^{1/16}E^{3/16}$ where e is the child's mother's education level and E is the average educational level of other women. Let there be one thousand women in the community. Note the child just takes x , e , and E as given.
 - (a) Is there a positive externality in consumption of e ? How does it operate? Do you expect that this externality will be internalized in any way? Intuitively (no math), what is the difference between the free-market e and the socially optimal levels?
 - (b) Suppose the price of x is $p_x = 1$ and the price of e is p_e . What is the woman's MRS in (x, e) space? If the woman's income is 2,000, what is her private demand curve for education?
 - (c) Suppose a social planner cares equally about women and children and that each woman has exactly three children. What is a social planner's MRS in (x, e) space? If the the a typical woman's income is 2,000, what is the social demand curve for education? Graph the two curves. What Pigouvian subsidy would correct the externality?

2. *Obesity*. This problem asks you to analyze the obesity epidemic using methods developed in Dupor and Liu, "Jealousy and Equilibrium Overconsumption," *American Economic Review*, March 2003.

- (a) Suppose that utility for food consumed (c), the typical American's food consumption (x), and labor (n) is given by

$$U(c, x, n) = \left(\frac{c^{1/2} - \frac{1}{2}x^{1/2}}{\frac{1}{2}} \right)^{\frac{3}{2}} - n$$

Assume that $c > 0.25x$. If "jealousy" is defined as $U_2 < 0$, show whether this utility function exhibits jealousy.

- (b) Suppose that the labor needed to buy food is given by $c = wn$ where w is some parameter representing the wage. Set up the consumer's utility maximization choice of c using the method of substitution. What is the first order condition?
- (c) What is the marginal rate of substitution in (n, c) space? (Note, this is extremely easy once you've done part (b).) If the wage rises (w goes up), how does this change the first order condition and the MRS? What happens to consumers' food consumption?
- (d) Write down the maximization problem and first order condition for a social planner. At the symmetric solution to (b), where $c = x$, does the planner give people as much food to consume as they would choose by themselves? Why or why not?

3. *LibraryTax*. There are 10,000 residents of a town, each of whom has an income of \$50,000. There is a numeraire good x and a library which is a public good; the number of books in the library is L . Each resident has a Cobb-Douglas utility function

$$u(x, L) = x^{0.99} L^{0.01}$$

The library can buy books for \$20 each.

- (a) What is the demand curve for library books of each resident of the town? How many books would be purchased if the library were financed by private contributions?
- (b) What is the social demand curve for library books?
- (c) Suppose the town sets a lump-sum tax t on each resident, so income is $50,000 - t$. If this tax is used to fund the library, what is the socially optimal tax revenue. How many books are purchased? (Note that since all residents are identical, we can assume that the optimal tax is the same on all of them.)