ECON 301, Professor Hogendorn

Problem Set 6

1. Do problem 11.6 parts a, b, and c from the Nicholson reading.

2. *SilvermanBaggs.* After Wesleyan, you take a job with the investment banking firm Silverman Baggs. It turns out that they employ two types of labor: cowboys (k) and geeks (g). These two types work together to produce output according to the following production function:

$$y = f(k, g) = kg + ak$$

   where y is output and a is a parameter. (Cowboys make deals, while geeks do analysis.)

   (a) Without actually finding the cost curves, does this production function exhibit increasing, decreasing, or constant returns to scale?

   (b) What are the marginal products of both types of labor? Discuss what would happen to them if a increased.

   (c) Write a one-paragraph story describing the work environment that causes the production function to have this form, and describe what might cause an increase in a.

   (d) Use the Lagrangian to solve for the total cost function (let wages of the two types be w_k and w_g.)

3. *Sigma.* Consider the production function

$$f(L, K) = (L^{0.25} + K^{0.25})^3$$

   (a) What is the formula for MRTS?

(b) Does this production function exhibit decreasing, constant, or increasing returns to scale?

(c) What is the elasticity of substitution?

4. *Tequila.* The spirit tequila is produced by distilling the fermented juice of the agave plant. True tequila can only be made from agave grown in the officially denominated tequila region in the environs of Tequila, Mexico. An agave plant takes 8 years to reach maturity, and the region was hit by a freak frost in 1997 that killed many agave plants.

   Suppose that tequila is produced according to the following production function:

$$f(A, X) = 143AX^{0.05}$$

   A is metric tons of agave and X is a composite factor including labor, oak casks, grinding equipment, and so forth. The idea behind this production function is that if $X = 1$, each ton of agave produces 143 liters of tequila, but X could be adjusted to change this amount.

   Let the price of X be 400 pesos and the price of a metric ton of agave is 1,000 pesos.

   (a) What is the long-run cost curve for tequila?

   (b) If distillers set LRAC(y)=6, how much A do they use?

   (c) Suppose that after the freeze, 90% of the amount of A from part (b) remains available, and so it becomes a fixed factor. If, nevertheless, distillers want to produce the same ouput, what is the cost?

Review problems only, not to turn in:

5. Do problem 11.1 from the Nicholson reading.

6. LRSR. A firm has the following production function:

$$f(K, L) = \sqrt{K} + \sqrt{L}$$

where K is capital and L is labor. The price of K is w_K and the price of L is w_L.

   (a) Use the Lagrangian to confirm that the long-run conditional factor demand functions for K and L are

$$K(w_K, w_L, y) = \frac{y^2}{\left(1 + \frac{w_K}{w_L}\right)^2}$$

$$L(w_L, w_K, y) = \frac{y^2}{\left(1 + \frac{w_L}{w_K}\right)^2}$$

   (b) If $w_K = 1$, $y = 90$, and $w_L = 2$ what is the LRMC?

   (c) Suppose that $y = 90$ happens to be the output at which LRAC is equal to SRAC. Assume capital is fixed in the short run and labor is variable, and continue to assume that $w_K = 1$ and $w_L = 2$. What is SRMC when $y = 120$?

   (d) Draw a graph of the LRAC, LRMC, SRAC, SRMC.

7. *VisaDiscover*. Visa and Discover are considering the introduction of debit cards. Both firms have the same production function $f(L, K) = L^{.8}K^{.3}$. Labor and capital both cost \$10 per unit.

   (a) What is the long run total cost curve for either company? Use the Lagrangian to show your answer.

   (b) Assume K is fixed in the short run. Confirm that the short-run total cost curve is $TC(y|K) = 10K + 10K^{-0.375}y^{1.25}$.

Answers to Review Problems:

5. *Nicholson11.1_a*

   (a) The graph is upward sloping and concave.

   (b) $AP_L = \frac{100\sqrt{L}}{L} = 100L^{-0.5}$. This is clearly decreasing in L.

   (c) $\frac{dq}{dL} = 100 \cdot 0.5L^{-0.5} = \frac{50}{\sqrt{L}}$. Because the total product of L is concave everywhere, it is always the case that MP_L is diminishing. That means that each additional worker always pulls down the average product.

6. LRSR_a.

   (a) The Lagrangian is:

$$\begin{aligned}
\min_{K,L,\lambda} \mathcal{L} &= w_K K + w_L L - \lambda(\sqrt{K} + \sqrt{L} - y) \\
\frac{\partial \mathcal{L}}{\partial K} &= w_K - \lambda \frac{1}{2} \frac{1}{\sqrt{K}} = 0 \\
\frac{\partial \mathcal{L}}{\partial L} &= w_L - \lambda \frac{1}{2} \frac{1}{\sqrt{L}} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= \sqrt{K} + \sqrt{L} - y = 0
\end{aligned}$$

   Solving simultaneously we get:

$$\lambda = 2w_K\sqrt{K}$$

$$\lambda = 2w_L\sqrt{L} \qquad \lambda = \lambda \Rightarrow 2w_K\sqrt{K} = 2w_L\sqrt{L}$$

$$\Rightarrow K = \frac{w_L^2 L}{w_K^2}$$

$$\sqrt{\frac{w_L^2 L}{w_K^2}} + \sqrt{L} - y = 0 \quad \Rightarrow \quad L(w_L, w_K, y) = \frac{y^2}{\left(1 + \frac{w_L}{w_K}\right)^2}$$

$$K(w_K, w_L, y) = \frac{y^2}{\left(1 + \frac{w_K}{w_L}\right)^2}$$

(b)

$$
\begin{aligned}
c(y) &= 1 \cdot K(1,2,y) + 2 \cdot L(2,1,y) \\
c(y) &= 1 \cdot \frac{y^2}{2.25} + 2 \cdot \frac{y^2}{9} \\
MC(y) &= 2 \cdot \frac{y}{2.25} + 4 \cdot \frac{y}{9} \\
MC(90) &= 80 + 40 = 120
\end{aligned}
$$

(c) $K(1,2,90) = 3600$ is the amount of capital available. Then the short run production function is:

$$f(3600, L) = 60 + \sqrt{L} \Rightarrow L = (y - 60)^2$$

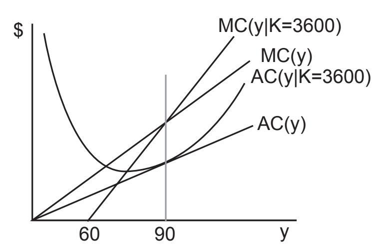
The short run total cost curve is:

$$c(y|K = 3600) = 3600 + 2(y - 60)^2$$

The short run marginal cost curve is:

$$MC(y|K = 3600) = 4(y{-}60) \quad \text{so} \quad MC(120|K = 3600) = 240$$

(d) A generic graph would look like Figure 21.10 on pg. 371 of Varian. (But note that Varian's publisher is imprecise because the MC curves do not quite cut the AC curves at their lowest points.) Specifically for these cost functions, the graph is:



Note that when a firm is operating on the upward-sloping portion of LRAC, the marginal cost curves are above the average cost curves.

7. *VisaDiscover_a.*

(a)

$$\begin{aligned}
\max_{K,L,\lambda} \mathcal{L} &= 10K + 10L - \lambda(K^{0.3}L^{0.8} - y) \\
\frac{\partial \mathcal{L}}{\partial K} &= 10 - \lambda 0.3K^{-0.7}L^{0.8} = 0 \\
\frac{\partial \mathcal{L}}{\partial L} &= 10 - \lambda 0.8K^{0.3}L^{-0.2} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= K^{0.3}L^{0.8} - y = 0
\end{aligned}$$

Solving simultaneously we get:

$$\begin{aligned}
\lambda = 33.33K^{0.7}L^{-0.8} & \\
\lambda = 12.5K^{-0.3}L^{0.2} & \qquad K = 0.375L \\
(0.375L)^{0.3}L^{0.8} - y = 0 & \;\Rightarrow\; 0.75L^{1.1} - y = 0 \\
L^* = 1.3y^{0.91} & \qquad K^* = 0.5y^{0.91}
\end{aligned}$$

$$TC(y) = 10K^* + 10L^* = 13y^{0.91} + 5y^{0.91} = 18y^{0.91}$$

(b)

$$y = K^{0.3}L^{0.8} \Rightarrow L^{0.8} = K^{-0.3}y \Rightarrow L(y|K) = K^{-0.375}y^{1.25}$$

$$TC(y|K) = 10K + 10L(y|K) = 10K + 10K^{-0.375}y^{1.25}$$