ECON 301, Professor Hogendorn

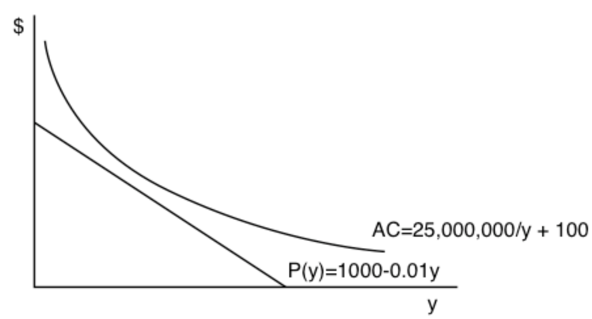
Problem Set 8

1. *Coke.* Suppose that all around the world, there are small towns in which the price elasticity of demand for Coca-cola is constant at -1.2. Each of these towns is served by a monopoly Coke distributor. However, the technology for distributing Coke varies widely: huge bottling plants and 18-wheeler truck delivery in the USA, local bottlers and van delivery in Japan, delivery by pack mule to isolated parts of Bolivia, etc.

   (a) What is the markup (Lerner Index) on Coke in these markets?

   (b) Let the production function be $f(K) = \beta K^2$, where β varies from place to place, and let the price of capital be 20. How does the price of Coke vary with β? (This is pretty tricky. Note that there is a constant elasticity demand.)

2. *WaterMonopoly.* The government has offered to give you a monopoly if you will provide water in a city of 100,000 people. The demand and average cost curves are shown in the following graph:



Assume that the inverse demand curve comes from 100,000 people who all have the same individual demand curve: $x(p) = 1 - 0.001p$.

(a) Using the graph, explain why the government must allow you to charge a two-part tariff if you are going to supply water.

(b) If the government will allow you to charge one price only, what is your profit or loss if you provide this service?

(c) If the government will allow you to charge a two-part tariff, what is your profit or loss if you provide this service?

3. Tractors. Two American companies, Case and John Deere, have decided to introduce their tractors in either the Polish market or the Hungarian market. Neither company has sufficient resources to enter both markets.

   If they both enter the Polish market, they both expect profits of $1 million. If they both enter the Hugarian market, they both expect profits of $1.5 million.

   If Case enters the Polish market and John Deere enters the Hugarian market, then Case expects profits of $3 million and John Deere expects profits of $4 million.

   If Case enter the Hungarian market and John Deere enters the Polish market, then Case expects profits of $5 million and John Deere expects profits of $3 million.

   There is a single consulting firm with special expertise that will enable either Case or John Deere to move first. The firm will offer its services to the highest bidder.

   Using a normal form game, describe what is the most likely outcome.

   (a) Case outbids John Deere for the consultant's services. Case enters the Polish market first and then John Deere enters the Hungarian market.

   (b) Case outbids John Deere for the consultant's services. Case enters the Hungarian market first and then John Deere enters the Polish market.

(c) John Deere outbids Case for the consultant's services. John Deere enters the Polish market first and then Case enters the Hungarian market.

(d) John Deere outbids Case for the consultant's services. John Deere enters the Hungarian market first and then Case enters the Polish market.

4. Normal. Find the Nash equilibrium(a) in the following normal form game:

	L	C	R
T	(2,2)	(5,0)	(1,1)
M	(0,5)	(4,4)	(1,1)
B	(1,1)	(1,1)	(2,2)

Review problems only, not to turn in:

5. *Technologies.* Suppose there are three technologies for providing a new Internet service, and one of them will eventually emerge as clearly superior to the other two. All three technologies use the factors S for servers and B for bandwidth, and the factor prices are $w_S = w_B = 1$.

   Technology A has production function $F(S, B) = S^{0.7}B^{0.7}$ but S must be set to 10 units and can never be changed.

   Technology B has production function $F(S, B) = S^{0.6}B^{0.6}$ and both factors can be freely varied.

   Technology C has production function $F(S, B) = S^{0.3}B^{0.6}$ and both factors can be freely varied.

   Recall that in a natural monopoly, $y^{MES} > y^{AC}$. If demand is $P(X) = 50 - 2X$, which of the above technologies would result in a natural monopoly? Do as little work as possible to answer, but explain your reasoning. Hint: For many cost functions, you can only find y_{AC} by using a computer (or a lot of trial and error). But you can easily solve this problem without resorting to these techniques.

6. *Consulting.* Technology can improve labor productivity. One might be concerned that this could be bad for workers since fewer would be needed to produce the same output. Displaced workers might have to move to another industry. To think about this, suppose an industry has the production function $f(L) = \alpha L^{0.5}$. The conditional factor demand is thus $L(y) = \left(\frac{1}{\alpha}\right)^2 y^2$. Let $w_L = 1$ throughout this whole problem (i.e. overall labor market equilibrium is unaffected by the changes in the industry we examine here). Suppose there is a fixed cost to start a firm which is $F = 2500$. The cost function is thus

$$c(y) = \left(\frac{1}{\alpha}\right)^2 y^2 + 2500$$

Note that this is both the short-run and the long-run cost function; the only difference is that in the long run a firm can exit or enter the industry.

Suppose that demand in the industry is given by $X(p) = 60000p^{\epsilon}$. Elasticity ϵ can take on two values: -0.5 and -1.5. Answer the following for each of these values:

(a) Suppose that initially $\alpha = 100$ and the industry is in long-run perfectly competitive equilibrium. How many firms are there? What is the total number of workers?

(b) Suppose that improved technology causes a change to $\alpha = 160$. In the short run (i.e. with the number of firms fixed) what is the new total number of workers?

(c) In the long run, the number of firms will adjust to the new situation. What is the new number of firms and the new total number of workers?

(d) Describe in words what an individual worker would experience during parts (a)-(c). For example, your description might read ``First I noticed that my firm hired a few new people.

Later, some of our competitors went out of business. Most of the people who worked for those firms came to work at my firm and our remaining competitors, but a few had to get jobs in another industry." Remember to do this for both values of elasticity, and discuss which elasticity is preferable for the workers.

7. Minus2. Suppose the demand curve for a good is:

$$x(p) = 1000p^{-2}$$

There is a monopoly which produces this good, and it has constant marginal cost of \$2 per unit.

   (a) What is the monopoly optimal price, quantity, and profit?

   (b) What is the deadweight loss of this monopoly?

Answers to Review Problems:

5. *Technologies_a.* Technology B must be a natural monopoly. The exponents add to more than 1, so there are economies of scale that are never exhausted. As a result, $y_{MES} \to \infty$ while y_{AC} is limited by the demand curve.

   Technology C could never be a natural monopoly. The exponents add to less than 1, so there are diseconomies of scale at all levels of output. As a result, $y_{MES} = 0$, while y_{AC} is positive.

   For Technology A, we can rewrite the production function as $F(B) = 5B^{0.7}$. Thus, $B = 0.1y^{10/7}$ and $c(y) = 10 + 0.1y^{10/7}$. The average cost is $AC(y) = \frac{10}{y} + 0.1y^{3/7}$. The marginal cost is $MC(y) = \frac{1}{7}y^{3/7}$. To find the MES,

$$\begin{aligned} AC(y) &= MC(y) \\ \frac{10}{y} + 0.1y^{3/7} &= \frac{1}{7}y^{3/7} \end{aligned}$$

$$\begin{aligned}\frac{10}{y} &= 0.04y^{3/7} \\ 10 &= 0.043y^{10/7} \\ y_{MES} &= 45.34\end{aligned}$$

Unfortunately, we can not explicitly solve for y_{AC}:

$$\begin{aligned}AC(y) &= P(y) \\ \frac{10}{y} + 0.1y^{3/7} &= 50 - 2y\end{aligned}$$

However, $P(45.34) = -40.68$, so it is clear that AC is declining over the entire economically meaningful portion of the demand curve. Therefore this technology does result in a natural monopoly.

6. *Consulting_a.*

   (a) The average and marginal cost curves in this case are:

   $$AC(y) = \frac{2500}{y} + 0.0001y \qquad\qquad MC(y) = 0.0002y$$

   Thus, the minimum average cost is $\frac{2500}{y} + 0.0001y = 0.0002y \Rightarrow y_{LR} = 5000$. At this output, the amount of labor employed by each firm is $L(5000) = 2500$.

   The marginal cost of this output level is $MC(5000) = 1$, and since perfectly competitive firms set price equal to marginal cost, we have $p_{LR} = 1$. This is the long run supply curve.

   Equating supply to demand, we find the demand at $p = 1$, which is $60000 \cdot 1^{\epsilon} = 60000$.

   The number of firms in the market must therefore be $N = \frac{60000}{5000} = 12$. Since each firm employs 2500 workers, total employment is 30000.

(b) Now that $\alpha = 160$, the conditional factor demand is $L(y) = 0.00004y^2$ and the total cost function is $c(y) = 0.00004y^2 + 2500$. Thus, the new marginal cost curve and the short-run firm supply curve is:

$$MC(y) = 0.00008y \qquad s(p) = 12500p$$

Since the number of firms cannot change in the short run, there are still 12 of them, so the market supply curve is just $12s(p)$, and setting supply equal to demand gives us:

$$\begin{aligned} 60000p^{\epsilon} &= 12 \cdot 12500p \\ p^{\epsilon-1} &= 2.5 \\ p &= 2.5^{\frac{1}{\epsilon-1}} \end{aligned}$$

$$p = (0.54, 0.69) \text{ when } \epsilon = (-0.5, -1.5)$$
$$X(p) = (81650, 104683) \text{ when } \epsilon = (-0.5, -1.5)$$
$$y = (6804, 8724) \text{ when } \epsilon = (-0.5, -1.5)$$
$$L(y) = (1852, 3044) \text{ when } \epsilon = (-0.5, -1.5)$$
$$NL(y) = (22224, 36528) \text{ when } \epsilon = (-0.5, -1.5)$$

(c) With $\alpha = 160$, the average and marginal cost curves are:

$$AC(y) = \frac{2500}{y} + 0.00004y \qquad MC(y) = 0.00008y$$

Thus, the minimum average cost is $\frac{2500}{y} + 0.00004y = 0.00008y \Rightarrow y_{LR} = 7906$. At this output, the amount of labor employed by each firm is $L(7906) = 2500$. (Note this is the same as before, which occurs because we have only changed the coefficient on the production function.)

The marginal cost of this output level is $MC(7906) = 0.632$, and since perfectly competitive firms set price equal to marginal cost, we have $p_{LR} = 0.632$. This is the long run supply curve.

Equating supply to demand, we find:

$$p = (0.632, 0.632) \text{ when } \epsilon = (-0.5, -1.5)$$
$$X(p) = (75473, 119420) \text{ when } \epsilon = (-0.5, -1.5)$$
$$N = (9.54, 15.1) \text{ when } \epsilon = (-0.5, -1.5)$$
$$L(y) = (2500, 2500) \text{ when } \epsilon = (-0.5, -1.5)$$
$$NL(y) = (23866, 37750) \text{ when } \epsilon = (-0.5, -1.5)$$

(d) Case of $\epsilon = -0.5$: I never should have taken the job at Sprint. Everything was fine until stupid researchers at Bell Labs and Nortel introduced the new technology. There was overcapacity everywhere, and Sprint laid off about 25% of its workforce. After a while, Global Crossing, Williams, and Worldcom filed for bankruptcy. But now that there's been some consolidation, Sprint is doing a little better, and it looks like the laid-off workers will be rehired. My friends at Global Crossing are out of luck though -- there won't be any telecoms jobs for them.

Case of $\epsilon = -1.5$: When I started at Nortel, it seemed like a sleepy firm, but then this great new technology came along. Nortel grew really fast, and we hired all kinds of new people. Fortunately, I saw that the good times couldn't last, so I cashed in my stock options and moved to a startup. It's a good thing, because Nortel laid off most of the people it hired. My new firm's hanging in there, but it's not like the boom times.

Clearly, the $\epsilon = -1.5$ is preferable, but note that even then there were some layoffs in this model. Also note that both examples provide a somewhat reasonable explanation of recent events in the telecoms industry, so it's hard to decide between the parameter values without delving deeper into the data and the model.

7. Minus2_a.

(a) This is easy because we have a constant elasticity demand curve with $\epsilon = -2$ and a constant marginal cost of \$2. Thus, the Lerner Index form of the monopoly's first order condition tells us that

$$\frac{p-2}{p} = -\frac{1}{-2} \Rightarrow p = 4$$

The demand curve tells us that $X(4) = 1000 \cdot 4^{-2} = 62.5$. The constant MC is the same as the AC, so there is a profit of \$2 per unit, or a total profit of 125.

(b) At $P = MC = 2$, the monopoly quantity is $X(2) = 1000 \cdot 2^{-2} = 250$. The deadweight loss is the area between the price of 2 and 4, but not including the monopoly profit:

$$\int_2^4 1000p^{-2}dp - 125 = -1000 \cdot 4^{-1} + 1000 \cdot 2^{-1} - 125 = \$125$$

This is represented by areas A and B in the following figure: