

ECON 301, Professor Hogendorn

Problem Set 9

1. *Varian* 27.1. Varian, Chapter 27, Review Question #1.
2. *Laspeyres*. Let your utility function be

$$u(X, Y) = \sqrt{X} + \sqrt{Y}$$

- (a) Use the Lagrangian to find the demand functions for X and Y and the value of λ .
 - (b) If $p_x = 1$, $m = 100,000$, and p_y rises from \$1 to \$1.20, what is the substitution effect and the income effect?
 - (c) Suppose that in response to the price change, you were given a raise based on a Laspeyres price index. What is the calculus approximation of your increase in utility? (Hint, use λ .)
3. *Aisha*. Aisha runs a one-person, ten-cow dairy operation which produces 600 gallons of milk a week. This is her sole source of income. Aisha's utility function is

$$U(x, g) = 60x^2g^4$$

where x = numeraire and g = gallons of milk. Let p_g be the price of milk.

- (a) What is Aisha's demand function for milk?
- (b) Show whether milk is a normal or an inferior good.
- (c) The price of milk is \$4 per gallon. How many gallons of milk does Aisha consume? How much numeraire?

- (d) All the dairies except Aisha's are hit by a tornado, wiping out many cows and causing the price of milk to rise to \$10 per gallon. Break down the corresponding change in Aisha's consumption of milk between the substitution and full income effects.

Review problems only, not to turn in:

4. *KmartWalMart*. Suppose that Kmart and Wal-Mart both produce a composite output q which is some measure of floorspace and sales. Kmart's and Wal-Mart's cost curves are

$$c(q_K) = q_K \quad c(q_W) = 0.7q_W$$

The market demand for the composite good is $p(Q) = 500 - 4(q_K + q_W)$. The firms are Cournot competitors. What is the price and what are Kmart's and Wal-Mart's market shares and profits?

5. *BigMacs*. You buy a lot of Big Macs. You are also on your town's zoning board, and McDonald's REALLY wants to build a new restaurant there. McDonald's raises the price of Big Macs from \$3 to \$4. Your demand for Big Macs is $x(p, m) = 0.01 \frac{m}{p^{1.5}}$. Your income m is \$50,000.

You complain about the price increase, and subtly hint that it could affect your zoning decision. In response, McDonald's sends a representative who will compensate you with coupons for free Big Macs (fractional coupons are allowed). Here are three possible ways to compensate you:

- (a) Calculate a Laspeyres price index, calculate the additional income you would need according to the price index, and divide that amount by \$4 to get the number of Big Mac coupons.
- (b) Use Slutsky income-compensated demand to calculate the substitution effect, and give that many Big Mac coupons.

- (c) Use Marshallian demand to calculate the change in demand, and give that many Big Mac coupons.

6. *Kadyrzhanova*. This problem is taken from Dalida Kadyrzhanova's Ph.D. thesis at Columbia University. Her idea is based on the notion that executives of firms tend to overweight revenues in their business decisions. This is because higher revenues mean a bigger firm which means more power and prestige for the executives.

Suppose there are two firms in a Cournot duopoly. Inverse demand is $p(Q) = 10 - 2Q$ where $Q = q_1 + q_2$ is the combined output of the two firms. The firms have identical, constant marginal costs $MC(q_i) = 1$. The chief executive of firm 1 maximizes a weighted average of α_1 times the profit of the firm plus $1 - \alpha_1$ times the revenue of the firm. The chief executive of firm 2 does the same with a separate parameter α_2 , not necessarily the same as α_1 .

- (a) Verify that the Cournot equilibrium quantities as functions of the parameters α_1 and α_2 are:

$$q_1 = \frac{5 - \alpha_1 + \frac{1}{2}\alpha_2}{3} \quad q_2 = \frac{5 - \alpha_2 + \frac{1}{2}\alpha_1}{3}$$

- (b) Find the Cournot profits (just the actual profits, not the weighted average the executives are maximizing) for the firms. What is the effect of a decrease in just α_1 on the Cournot quantities *and* Cournot profits of firm 1 and firm 2?
- (c) Using intuitive arguments rather than referring to the algebra, why does α_1 have the effect that it does? Think carefully.
- (d) Suppose that prior to playing the Cournot game described above, the boards of directors of the two firms play a simultaneous normal form game. Firm 1's board can choose $\alpha_1 = 1$ or $\alpha_1 = 2/3$. And firm 2's board can choose $\alpha_2 = 1$ or

$\alpha_2 = 2/3$. Write the normal form of this game, deriving the payoffs in each case from the profit functions you found in part (a). What is the Nash equilibrium?

(e) Describe in words the logic of the Nash equilibrium in (d).

Answer to Review Problems 4 and 5:

4. *KmartWalMart_a*. In this case, Wal-Mart has $MC_W = 0.9$ while Kmart has $MC_K = 1$. We can save time by solving the Cournot problem in its general form for a firm 1 with marginal cost MC that has a rival firm 2:

$$\max_{y_1} = (500 - 4(y_1 + y_2))y_1 - MCy_1$$

$$\text{FOC: } 500 - 4y_1 - 4y_2 - 4y_1 = MC$$

$$500 - 8y_1 - 4y_2 = MC$$

$$8y_1 = 500 - MC - 4y_2$$

$$y_1 = 62.5 - \frac{MC}{8} - \frac{y_2}{2}$$

From this we see that

$$y_K(y_W) = 62.375 - \frac{y_W}{2} \quad y_W(y_K) = 62.3875 - \frac{y_K}{2}$$

Now solving simultaneously to find a Cournot-Nash equilibrium:

$$y_K = 62.375 - \frac{62.3875 - \frac{y_K}{2}}{2}$$

$$y_K = 62.375 - 31.19375 + \frac{y_K}{4}$$

$$y_K \left(1 - \frac{1}{4}\right) = 31.18125$$

$$y_K = 41.575$$

And plugging this back into the y_W function gives

$$y_W = 62.3875 - \frac{41.575}{2} = 41.6$$

At these outputs, the price is $P(41.575 + 41.6) = 167.3$, Kmart's profit is $(167.3 - 1)41.575 = 6913.93$ and Wal-Mart's profit is $(167.3 - 0.9)41.6 = 6918.08$. The market shares are $41.575/83.175 = 49.98\%$ for Kmart and $41.6/83.175 = 50.02\%$ for Wal-Mart.

5. *BigMacs_a*.

(a) Laspeyres price index

$$\frac{p_{x2}x_1 + p_{y2}y_2}{p_{x1}x_1 + p_{y1}y_1} = \frac{4(96.23) + 49711.31}{3(96.23) + 49711.31} = 1.0019246$$

$$1.0019246 * 50000 = 50096.23$$

$$50096.23 - 50000 = 96.23$$

$$96.23/4 = 24.0575 \text{ coupons}$$

(b) Substitution effect:

We already saw that the Laspeyres price index adds 96.23 to income; this is the same as Slutsky income compensation.

$$x(4, 50096.23) - x(3, 50000) = 62.62 - 96.23 = -33.61$$

So this method gives 33.61 coupons.

(c) Marshallian demand:

$$x(4, 50000) - x(3, 50000) = 62.50 - 96.23 = -33.73$$

This method gives 33.73. It's a little more, but not surprisingly there is almost no income effect here because the share of income spent on Big Macs is still very small despite your gluttony.