ECON 301, Professor Hogendorn

Review Problem Set 10

1. *12firms.* There is a firm with production function

$$q = f(L, K) = L^{1/2} + K^{1/2}$$

This firm is initially stuck in the short run with $\overline{K} = 16$ which cannot be changed. The wage is $w = 3$ and the price of capital is $r = 4$.

   (a) Find the short run marginal cost curve and the short-run supply curve.

   (b) If there are 12 firms, and if market demand is $\mathrm{q(p)} = 96 - \mathrm{p}$, what is the short-run market equilibrium price?

   (c) What is the short-run average total cost? Is this firm making a loss, breaking even, or making a super-normal profit? Illustrate on a two-panel graph, one panel showing the market, the other showing the cost curves of an individual firm.

2. *Glendivot.* The Glendivot distillery makes Scotch whisky and then stores it for a period of time before selling it. The market price of a barrel of Glendivot is

$$V(t) = 480t - 12t^2$$

where t is the number of years of aging.

   (a) The distillery produces a private reserve which is consumed only by the owner's family. The objective is to produce the highest possible value Scotch without reference to costs. How long is the private reserve aged?

(b) The distillery also produces its regular product for commercial sale. Its faces an interest rate of 5%, compounded continuously. How many years does it age its regular product?

(c) Suppose the market for Scotch is perfectly competitive and in long run equilibrium with no entry or exit of firms (a dubious assumption, but let's go with it). Assume there is only one cost the distillery incurs: the cost of distilling a barrel's worth in the first place. How much does it cost to distill a barrel? (Note: ignore the private reserve; that is private consumption by the owner.)

3. *Mackenzie.* Mackenzie Rembrandt had a famous ancestor who painted a family portrait that has been handed down to her. The value of this painting increases over time according to the function

$$V(t) = 10t - 2t^2$$

where t is the number of years starting now and V is measured in millions of dollars.

(a) If Mackenzie's objective is to let the painting appreciate to its maximum value and then give it to a museum, when should she give away the painting?

(b) Suppose instead that Mackenzie views the painting as an investment, and she can get a (continuously compounded) interest rate of 7%. When should she sell the painting?

4. *Meat-packing.* Let the demand in a city for meat be

$$\mathrm{Q} = 24 - 4\sqrt{\mathrm{p}}$$

There are 6 meat-packing plants, each with production function $\mathrm{f(L)} = \mathrm{L}^{1/3}$ where L is the number of hours that meat-packers are employed. The wage for meat-packers is $\mathrm{w} = 3$ never changes in this problem.

(a) Suppose all 6 meat packing plants are separately owned, and therefore we will treat them as perfect competitors. What is an individual firm's supply curve and the market supply curve?

(b) How many hours of meat-packing work will there be in market equilibrium?

(c) Now suppose all six plants are purchased by one company which operates them as a monopoly, allocating 1/6 of its total production Q to each plant. What is this monopoly's MC curve?

(d) Will there be more or fewer hours of meat-packing work at the monopoly profit max? (You can get full credit for justifying your answer with a monopoly diagram rather than actually calculating the answer. But I will give you 1 point of extra credit for actually finding the number of hours. You can use Wolfram Alpha to help if you want. Don't do this until you're done with everything else!)

Answers:

5. *12firms_a.*

(a) Since $q = L^{1/2} + 4$, the short-run conditional factor demand for labor is

$$L^{1/2} = q - 4 \Rightarrow L(q) = (q-4)^2$$

Short run total cost $TC(q) = wL(q) + r\overline{K} = 3(q-4)^2 + 64$. Then marginal cost is

$$MC(q) = \frac{dTC(q)}{dq} = 6(q-4) = 6q - 24$$

A perfectly competitive firm sets MC=p, so its supply is

$$p = 6q - 24 \Rightarrow q = \frac{p+24}{6} \Rightarrow s(p) = 4 + \frac{1}{6}p$$

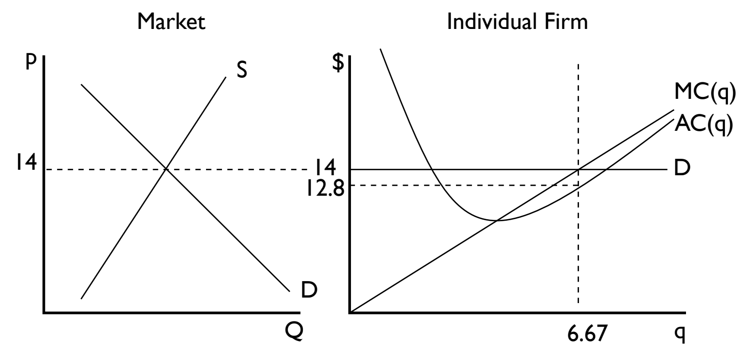
(b) With 12 firms, market supply equals market demand is

$$12s(p) = q(p) \Rightarrow 48 + 2p = 96 - p \Rightarrow p^* = 16$$

(c) At $p = 16$, each individual firm produces $s(16) = 6.67$. Short run average cost is

$$AC(q) = \frac{TC(q)}{q} = \frac{3(q - 4^2)}{q} + \frac{64}{q}$$

so AC(6.67) = 3.2 + 9.6 = 12.8. Since this is lower than the price of 16, the firm makes a profit.



6. *Glendivot_a.*

(a) To solve $\max_t V(t) = 480t - 12t^2$ find the FOC:

$$V'(t) = 480 - 24t = 0 \Rightarrow t = \frac{480}{24} = 20$$

(b) To solve $\max_t PV(V(t)) = e^{-.05t}(480t - 12t^2)$ the FOC is:

$$\begin{aligned} \tfrac{dPV(V(t))}{dt} &= (480 - 24t)e^{-.05t} - .05e^{-.05t}(480t - 12t^2) = 0 \\ 480 - 24t - 24t + 0.6t^2 &= 0 \\ 0.6t^2 - 48t + 480 &= 0 \end{aligned}$$

Use the quadratic formula or a calculator to find that $t = 11.7$.

(c) Using the best ``technology'' available (i.e. $t = 11.7$), there must be a present value of zero when discounted at the proper rate:

$$PV = -c_B + e^{-.05 \cdot 11.7}(480 \cdot 11.7 - 12(11.7)^2) = 0$$
$$c_B = \$2,213.56$$

7. *Mackenzie_a.*

(a) Just maximizing the value of the painting leads to the first order condition:

$$\frac{dV}{dt} = 10 - 4t = 0 \Rightarrow t = 2.5 \text{ years}$$

(b) In this case, the problem is to maximize $\frac{V(t)}{e^{0.07t}}$, which leads to the first order condition:

$$(10 - 4t)e^{-0.07t} + (10t - 2t^2)(-0.07)e^{-0.07t} = 0$$

The exponential terms cancel, and we can simplfy to:

$$0.14t^2 - 4.7t + 10 = 0$$

Using a calculator (or the quadratic formula), we find

$$t = 2.28 \text{ or } 31.29$$

Only the first answer is economically meaningful in this situation.

(Answer to meat-packing will come later.)