ECON 301, Professor Hogendorn

Problem Set 2

1. *StableChina*. On pg. 232 of his 1990 book *The Lever of Riches*, economic historian Joel Mokyr discusses the slowdown in Chinese versus European technological change after 1400:

   > ... technological progress is a positive-sum game, with winners and losers. Although by definition total gains exceed total losses, the adjustment costs and possible political unrest may have constituted a price that some societies were not willing to pay. The evaluation of the social costs of technological progress is difficult; they could differ immensely from place to place. What may have appeared as a very cheap lunch in the West may have appeared as unacceptably costly in China. A decline in the rate of technological change in China could thus be attributed to a change in social preferences in the direction suggested by [Hsia-tung] Fei ([*China's Gentry*, University of Chicago Press,] 1953, pg. 74), who emphasized the desire of Chinese society to avoid the social conflicts often entailed by technological changes.

   Mokyr gives other lines of argument, but this seems to be the one he finds most convincing, so let's analyze it. Assume that there is a tradeoff between two goods, stability (S) and growth (G), and that initially both China and Europe are at the point $(S, G) = (100, 100)$. Assume that we can write down utility functions for the whole society (this is a common assumption in political economy) which are:

   $$u_C(S, G) = S^a G^{1-a} \qquad u_W(S, G) = S^b G^{1-b}$$

for China and the West respectively.

(a) At the $(S, G) = (100, 100)$ point, China has an MRS of 2 units of growth per 1 unit of stability and the West has an MRS of one unit of growth per 5 units of stability. Graph the two societies' indifference curves through the $(100, 100)$ point.

(b) Find a and b.

(c) Suppose that the aristocracy in both regions is contemplating a change to (101,99), i.e. a change in favor of stability but sacrificing growth. Use differentials to show the change in utility in both regions.

(d) Suppose we discovered that each region could trade along a budget line connecting the points (0,200) and (200,0). Suppose we then solved the utility maximization problem for both China and the West by using the Lagrangian. In words, what would be the interpretation of λ?

2. *Electricity*. Suppose you want to study electricity demand. You might conclude that most families' tradeoff between e (electricity) and x (the numeraire) is independent of m. In this case, a quasilinear utility function is appropriate, along with a standard budget line.

$$\begin{aligned} u(e, x) &= w(e) + x \\ p_e e + x &= m \end{aligned}$$

(a) Set up the Lagrangian for this problem and take the first order conditions. (Note that $w(e)$ is some function you do not know explicitly).

(b) What is the demand function for e and the value for λ at the optimum?

(c) Explain what the value of λ means and why the particular value you found makes sense.

(d) Is there anything else you should check to make sure that the answer reasonable? Hint: what if income were very low?

3. *Medical2.* Prices of medical services have been rising much faster than other goods and services in the economy. Let μ be medical services and x be all other goods. Suppose that a consumer has a demand curve for medical services of

$$\mu(p_\mu, p_x, m) = \frac{m}{4.5p_\mu}$$

(a) In 2007, the prices were $p_x = 1$, $p_\mu = 1$, and $m = 54.5$. By 2011 prices had risen to $p'_x = 1.08$, $p'_\mu = 1.12$ and income had fallen to $m' = 50.1$. Draw an indifference curve diagram, (with x on the x-axis) showing the two budget lines and the two optimal points. Remember that all income not spent on μ is spent on x.

(b) Calculate the Laspeyres price index for the price change from 2007 to 2011.

(c) Calculate the Paasche price index for the price change from 2007 to 2011.

(d) If the consumer had been given a raise based on the Laspeyres price index, how much x and μ would she have consumed in 2011. Would her utility have been higher or lower than in 2007?

Review Problems, not to turn in:

4. *Martini.* The Martini is a famous cocktail that is properly made with gin and vermouth. (Vodka martinis are a horrible travesty from the 1960s and 70s.) Let your utility function be

$$u(G, V) = G^{0.9}V^{0.1}$$

where G is ounces of gin and V is ounces of vermouth. Let the price of gin be \$1 per ounce and the price of vermouth be \$0.40 per ounce.

(a) Using only the information above, describe what proportions you use to make a martini.

(b) Now suppose that you have \$4 available to spend on martinis. Use the Lagrangian to solve the utility maximization problem, and then show how many total ounces are in the drink you make.

(c) If you had one more dollar, how much additional utility would you receive?

5. Budget. There are two goods, both of which are provided by the government. One is defense and homeland security spending (d), and the other is all other government discretionary functions (g) (environmental protection, transportation, health and human services, etc., but not including social security, Medicare, or welfare). Both are measured in billions of dollars, so the price of each good is 1 billion.

   In President Bush's 2007 budget proposal, $d = 472$ and $g = 398$. If we take the sum of these two as given, we can call the government's ``income" $m = 870$.

   Consider a Senator with the utility function

$$u(g, d) = 2g + dg$$

   (a) If this Senator could decide the government budget allocation herself, what would she pick? Use the Lagrangian to show that $G^* = 436$ and $D^* = 434$.

   (b) Using the total differential of the utility function, estimate the change in utility of this Senator if she were able to change the budget to her preferred point.

(c) Draw an accurate graph of the two points, showing the budget line and indifference curves.

(d) Now suppose that the Senator would have to ``pay'' for the privilege of choosing the budget allocation by a reduction in m. (For example, the Senator might have to offer tax cuts to get her budget passed, and these tax cuts would reduce the m available.) Use the Lagrange multiplier to determine the largest reduction in m the Senator would tolerate.

(e) This Senator is fairly middle-of-the-road in political views. Suppose you want to model a more liberal Senator, one who would prefer much more g than d. Should you change the utility function to $u(G, D) = 0.0002G + DG$ or $u(G, D) = 200G + DG$. Explain your answer with reference to the concepts of marginal utility and marginal rate of substitution.

6. *Lambda.* Suppose that a consumer has utility function $u(x, y) = x^{1/3}y^{2/3}$. The income is m, and the prices of the goods are both 1. Use the Lagrangian to solve for the value of λ. Then find $\partial u(x^*, y^*)/\partial m$ where x^* and y^* are the optimal solutions that come out of the Lagrangian. What is the relationship between λ and $\partial u(x^*, y^*)/\partial m$?

Answers to Review Problems:

5. *Martini_a*

(a)

$$MRS = -\frac{\frac{\partial u}{\partial G}}{\frac{\partial u}{\partial V}} = -\frac{.9G^{-.1}V^{.1}}{.1G^{.9}V^{-.9}} = -9\frac{V}{G}$$

Now set MRS equal to the slope of the budget line:

$$-9\frac{V}{G} = -\frac{1}{.4} = -\frac{10}{4} \Rightarrow G = \frac{36}{10}V \Rightarrow G = 3.6V$$

So use 3.6 parts gin to one part vermouth.

(b)

$$\begin{aligned}
\max_{G,V,\lambda} \mathcal{L} &= G^{.9}V^{.1} - \lambda(G + .4V - 4) \\
\frac{\partial \mathcal{L}}{\partial G} &= .9G^{-.1}V^{.1} - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial a} &= .1G^{.9}V^{-.9} - .4\lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= G + .4V - 4 = 0
\end{aligned}$$

Solving simultaneously we get:

$\lambda = \lambda \Rightarrow G = 3.6V$ (We already found this in part a)

$3.6V + .4V - 4 = 0$

$4V = 4 \Rightarrow V = 1 \quad G = 3.6$

So you make a martini using 1 ounce of vermouth and 3.6 ounces of gin, so the total amount of liquid in the drink is 4.6 ounces.

(c) $\lambda = .9G^{-.1}V^{.1} = .9 \cdot 3.6^{-.1} \cdot 1 = .79$

6. *Budget_a.*

(a) The Langrangian is:

$$\begin{aligned}
\max_{d,g,\lambda} \mathcal{L} &= 2g + dg - \lambda(d + g - m) \\
\frac{\partial \mathcal{L}}{\partial g} &= 2 + d - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial d} &= g - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= d + g - m = 0
\end{aligned}$$

Solving simultaneously we get:

$$\lambda = \lambda \Rightarrow 2 + d = g$$

$$d + 2 + d - m = 0$$
$$d = \frac{m-2}{2}, g = \frac{m+2}{2}$$
$$\lambda = g = \frac{m+2}{2}$$

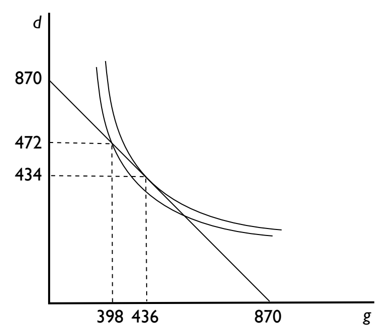
In this case, $m = 870$ so $g^* = 436$ and $d^* = 434$.

(b) The changes here are $dg = 436 - 398 = 38$ and $dd = 434 - 472 = -38$. Then the differential evaluated as Bush's point (the starting point) is:

$$du = \frac{\partial u}{\partial g} dg + \frac{\partial u}{\partial d} dd$$

$$du = (2 + d)dg + gdd = 474(38) + 398(-38) = 2888$$

(c) The key here is that Bush's point is not tangent to the Senator's indifference curve, and therefore the Senator must be getting less utility.



(d) We know from part (b) that the Senator gains 2,888 units of utility from getting the right to choose the allocation. From part (a), we know that $\lambda = g$, so at the Senator's optimum, $\lambda = 436$. Then the approximate reduction in m that the Senator would tolerate and still be indifferent is $dm = \frac{2{,}888}{\lambda} = 6.62$.

(e) The marginal utility of g is $MU_g = 0.0002 + d$ in the first function and $MU_g = 200 + d$ in the second. Since we except the liberal Senator to value g more highly, it makes sense to choose the second function.

The MRS for the first function is $MRS = \frac{0.0002+d}{g}$ while for the second function it is $MRS = \frac{200+d}{g}$. Thus the MRS is higher for the second function, meaning that for a one unit increase in g, the Senator would be willing to give up more d. Again, the second function is more consistent with the liberal Senator's preferences.

Note that technically speaking, we only need the MRS to rise because it's the one that measures the tradeoff. We actually could pick a function with a lower MU_g, but if it had a higher MRS it would still be more consistent with the liberal Senator's preferences.

7. *Lambda_a.*

$$\begin{aligned}
\max_{x,y,\lambda} \mathcal{L} &= x^{1/3}y^{2/3} - \lambda(x+y-m) \\
\frac{\partial \mathcal{L}}{\partial x} &= \frac{1}{3}x^{-2/3}y^{2/3} - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial y} &= \frac{2}{3}x^{1/3}y^{-1/3} - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= x+y-m = 0
\end{aligned}$$

Solving simultaneously we get:

$$\lambda = \lambda \Rightarrow y = 2x$$

$$x + 2x - m = 0$$

$$x = \frac{m}{3},\ y = \frac{2m}{3},\ \lambda = \frac{1}{3}x^{-2/3}y^{2/3} = 0.529$$

Now subbing the optimal x, y into the utility function gives:

$$U(x^*, y^*) = \frac{m}{3}^{1/3}\frac{2m}{3}^{2/3} = 0.529m$$

From this it is clear that $\partial u(x^*, y^*)\partial m = 0.529 = \lambda$.