ECON 301, Professor Hogendorn

Problem Set 2 Answers

1. *StableChina_a.*

   (a) S goes on the horizontal axis, G on the vertical. At the $(100, 100)$ point, a tangent line to China's curve has a slope of -2, while a tangent line to the West's curve has a slope of -0.2.

   (b) For China, we first find the MRS:

   $$MRS = -\frac{\frac{\partial u_C}{\partial S}}{\frac{\partial u_C}{\partial G}} = -\frac{aS^{a-1}G^{1-a}}{(1-a)S^aG^{-a}} = -\frac{a}{1-a}\frac{G}{S}$$

   Then find a such that the MRS is -2 at the $(100, 100)$ point:

   $$-\frac{a}{1-a}\frac{100}{100} = -2 \Rightarrow -a = -2 + 2a \Rightarrow a = 2/3$$

   Following the same procedure for the West gives $b = 1/6$.

   (c) First take the differential for China:

   $$du_c = \frac{\partial u_C}{\partial S}dS + \frac{\partial u_C}{\partial G}dG = aS^{a-1}G^{1-a}(+1) + (1-a)S^aG^{-a}(-1)$$

   Now plug in the current point, $S, G = 100$:

   $$du_c = a100^{a-1}100^{1-a}(+1) + (1-a)100^a100^{-a}(-1) = a - (1-a) = 2a - 1$$

   Thus China gains 1/3 in utility.

   Following the same procedure for the West gives a loss of -2/3.

   (d) In any Lagrangian problem, λ is the increase in the objective function when the constraint is relaxed by one unit.

In this case, the interpretation is that λ gives the additional societal utility if there were an opportunity to increase either growth or stability by one unit without having to make any tradeoff. Perhaps this would happen if there were excellent weather or new land were brought intro agricultural production.

2. Electricity_a.

(a) The Lagrangian is:

$$
\begin{aligned}
\max_{x,y,\lambda} \mathcal{L} &= w(e) + x - \lambda(x + p_e e - m) \\
\frac{\partial \mathcal{L}}{\partial x} &= 1 - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial e} &= w'(e) - \lambda p_e = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= x + p_e e - m = 0
\end{aligned}
$$

(b) Solving simultaneously we get:

$$
\begin{aligned}
\lambda &= 1 \\
w'(e) &= p_e \\
x &= m - p_e e
\end{aligned}
$$

To get the actual demand curve for e, we need to invert $w'(e)$, so

$$
e(p_e) = w'^{-1}(e)
$$

(c) In general, λ is the effect of relaxing the constraint, in this case the increase in utility form having 1 more dollar of income. For this quasilinear utility function, it makes sense that $\lambda = 1$ because one more dollar of income can buy one more unit of numeraire, and utility increases 1 for 1 in numeraire.

(d) Nothing about the FOC checks that you can actually afford the optimal e. So we still need to check that $p_e e^* \leq m$.
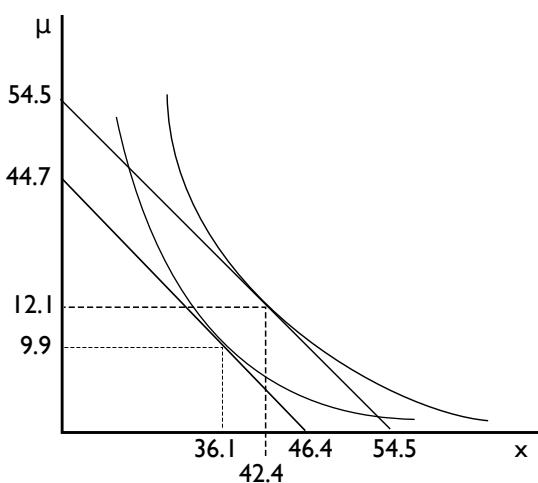
3. *Medical2_a.*

(a) In 2007,

$$\mu(1, 1, 54.5) = \frac{54.4}{4.5 \cdot 1} = 12.11, \qquad x = 54.5 - 1 \cdot 12.11 = 42.39$$

At the new prices in 2011,

$$\mu(1.12, 1.08, 50.1) = \frac{50.1}{4.5 \cdot 1.12} = 9.94, \qquad x = \frac{50.1 - 1.12 \cdot 9.94}{1.08} = 36.08$$

(b) Laspeyres:

$$\frac{1.08 \cdot 42.39 + 1.12 \cdot 12.11}{1 \cdot 42.39 + 1 \cdot 12.11} = 1.0889$$

(c) Paasche:

$$\frac{1.08 \cdot 36.08 + 1.12 \cdot 9.94}{1 \cdot 36.08 + 1 \cdot 9.94} = 1.0886$$

(d) The Laspeyres raise would give a new income of $m' = 54.5 \cdot 1.0889 = 59.34$. With that income, her consumption would

have been

$$\mu(1.12, 1.08, 59.34) = \frac{59.34}{4.5 \cdot 1.12} = 11.77$$

$$x = \frac{59.34 - 1.12 \cdot 11.77}{1.08} = 42.74$$

Utility must be higher, because this is a lump-sum raise based on the Laspeyres price index and the lump-sum principle says that any substitution that takes place can only raise utility. The household could still have afforded the old bundle, so the new one must give higher utility.

Just to make sure, you can double check that the new bundle, (42.74,11.77), would have cost 54.51 at the old prices, which is (very slightly) greater than the original income.