ECON 301, Professor Hogendorn

Problem Set 3

1. *Apartments.* Suppose the market demand curve for apartments is

$$x(p) = \left(100 - \frac{pm^2}{2880}\right)$$

x is the number of apartments rented, p is the monthly rent on a typical apartment, and m is the monthly income of a typical consumer in thousands of dollars.

(a) If the current rent is $900 and the current income is $4, graph the demand curve and the Engel curve, labeling the current point and the intercepts.

(b) Which of the following describe apartments at the current price and income: necessity, luxury, normal, inferior. (Presumably people would rather own houses or condos if they have higher incomes.)

(c) If the price falls to $800, what lump-sum tax or subsidy would leave the consumer able to purchase the same consumption bundle as before?

2. AishaMrLee. Aisha's utility function is

$$u(G, V) = G^{0.7}V^{0.3}$$

and Mr. Lee's utility function is

$$u(G, V) = G^{0.9}V^{0.1}$$

Aisha has 20 ounces of G and 10 ounces of V. Mr. Lee has 15 ounces of G and 15 ounces of V. This is all the G and V there is in the world, and there are no other people to trade with.

(a) Calculate the MRS in (G, V) space for both consumers at the endowment point.

(b) Draw an Edgeworth box showing the endowment and indifference curves of the consumers. (The indifference curves do not have to be plotted to match the utility function perfectly.)

(c) Assume that Aisha and Mr. Lee can trade at a market price as price-takers. If we set G as the numeraire, what is the price of V? What is the final allocation of G and V?

(d) Show the trading in your diagram.

Review Problems, not to turn in:

3. *Urp.* The residents of Urp consume only pork chops (X) and Coca-Cola (Y). The utility function of the typical resident of Urp is given by

$$U(X, Y) = \sqrt{XY}$$

In 2006, the price of pork chops in Urp was \$1 each; Cokes were also \$1 each. The typical resident consumed 40 pork chops and 40 Cokes (saving is impossible in Urp). In 2007, swine fever hit Urp, and pork chop prices rose to \$4; the Coke price remained unchanged. At these new prices, the typical Urp resident consumed 20 pork chops and 80 Cokes.

(a) What was the change in utility from 2006 to 2007? (Just plug into the utility function, don't use differentials.)

(b) What was the Laspeyres price index for 2007?

(c) What was the Paasche price index for 2007?

(d) What do you conclude about the ability of price indices to measure changes in welfare? (Hint: calculate how much income the typical Urp resident had in 2006 and 2007.)

4. <u>Pate.</u> There are two goods, goose liver pate (G) and beef (B). The typical French person has an endowment of $\omega_G = 50, \omega_B = 50$ and a utility function $U(G, B) = G^{0.7}B^{0.3}$. The typical American has an endowment of $\omega_G = 30, \omega_B = 70$ and a utility function $U(G, B) = B^{0.8}$. Note that the typical American simply does not receive utility from the pate.

   (a) What is the typical French and American MRS in (B, G) space at the endowment points?

   (b) Draw an Edgeworth box and show indifference curves for each type of consumer. Show the core and the contract curve.

5. *Pareto.* Is it possible to have a Pareto efficient allocation where someone is worse off than he is at an allocation that is not Pareto efficient? Illustrate with an Edgeworth Box.

Answers to Review Problems:

3. *Urp_a.*

   (a)

   $$
   \begin{aligned}
   U_{2006} &= \sqrt{40 \cdot 40} = 40 \\
   U_{2007} &= \sqrt{20 \cdot 80} = 40
   \end{aligned}
   $$

   (b)

   $$\frac{4 \cdot 40 + 1 \cdot 40}{1 \cdot 40 + 1 \cdot 40} = 2.5$$

   (c)

   $$\frac{4 \cdot 20 + 1 \cdot 80}{1 \cdot 20 + 1 \cdot 80} = 1.6$$

(d) We know that in actual fact, utility was unchanged and the new income in 2007 must have been $4 \cdot 20 + 1 \cdot 80 = 160$ which was twice the income of $1 \cdot 40 + 1 \cdot 40 = 80$ in 2000. Thus, Laspeyres overstated the amount of income needed to keep utility constant, and Paasche understated it.

4. Pate_a.

   (a) The French and American MRSs at the endowments are

$$\text{French: } -\frac{\frac{\partial U}{\partial B}}{\frac{\partial U}{\partial G}} = -\frac{0.3G^{0.7}B^{-0.7}}{0.7G^{-0.3}B^{0.3}} = -0.43\frac{G}{B} = -0.43$$

$$\text{American: } -\frac{\frac{\partial U}{\partial B}}{\frac{\partial U}{\partial G}} = -\frac{0.8B^{-0.2}}{0} = -\infty$$

   (b) The only tangency between the indifference curves is when the French person has $B = 0$. Thus the whole left side of the diagram is part of the contract curve. In addition, all allocations where the American has $G = 0$ are Pareto efficient, and are therefore also on the contract curve. The core is the portion of the contract curve along which the both the French person and the American gain more utility than their endowments. In any trading, we would expect the American to trade away all her endowment of G.
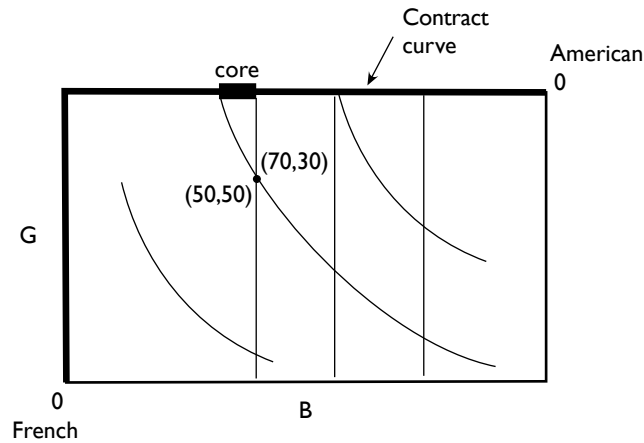
5. *Pareto_a.* Yes, Pareto efficiency says that it is not possible to make one person better off without making another person worse off. But that does not preclude making one person better off *and* making the the other worse off. For example, in the graph Vilfredo is better off at point B than point A, even though B is not on the contract curve and not Pareto-efficient while A is.