

ECON 301, Professor Hogendorn

Problem Set 4

1. *RichAndPoor*. A very rich person and a very poor person are going to trade in an Edgeworth box. The rich person is named Ms. 1 and her origin is the lower left corner. The poor person is named Mr. 2 and his origin is the upper right hand corner. The two people will trade good y (on the vertical axis) and good x (on the horizontal axis). Ms. 1 has the entire endowment of good x , and there is a lot of that good. Mr. 2 has the entire endowment of good y , but there is not that much of it. Both people's indifference curves indicate that good y doesn't bring very much utility compared to good x .
 - (a) Draw the Edgeworth box, showing the endowment point, indifference curves, and the contract curve. What is the Walrasian equilibrium? Is it efficient?
 - (b) Suppose the government values equality and wants the final outcome of trading to be the allocation approximately in the center of the box. Show a government price control that forces the center point to be in the budget sets of both consumers. How does this change the Walrasian equilibrium? Is "equality" achieved? Is this solution efficient.
 - (c) Can the government use the Second Fundamental Theorem of Welfare Economics to improve on part (b)?
2. *CokePepsi*. The income elasticity demand for Coke is $\epsilon_m^c = 0.58$. For Pepsi, the income elasticity is $\epsilon_m^p = 1.38$ at the current equilibrium points
 - (a) Which apply to Coke and Pepsi: normal, inferior, luxury, necessity? Why?

(b) Suppose 1 bottle of either drink costs 1/20 of a day's pay in a developing country, i.e. $p = m/20$. Draw the Engel Curves for Coke and Pepsi. What is the slope of each Engel Curve at the equilibrium point?

(c) Suppose we calculated a cross-price elasticity of Coke for Pepsi:

$$\epsilon_{cp} = \frac{\partial q_{\text{coke}}}{\partial p_{\text{pepsi}}} \frac{p_{\text{pepsi}}}{q_{\text{coke}}}$$

What sign do you expect? Why?

(d) Suppose the demand function for Coke is $q_{\text{coke}}(p_{\text{coke}}, p_{\text{pepsi}}, m)$. Write the total differential of this function.

3. *Laffer*. The "Laffer Curve" first became famous in the Reagan administration. It shows that when taxes are high enough, raising taxes can actually *reduce* tax revenue. While the theory is sound, its application to the U.S. income tax did not raise revenue under Reagan or either Bush.

Suppose there is a tax t so that consumer pay price p and sellers receive price $p - t$. Demand is $x(p)$, and supply is $s(p - t)$.

(a) Graph the tax and show the deadweight loss. Explain in words the deadweight loss.

(b) What is the change in equilibrium price when the tax changes? I.e., what is $\frac{dp}{dt}$? Express this in terms of elasticities as much as possible.

(c) Now since tax revenue is $TR = tx(p)$, find $\frac{dTR}{dt}$. Put it in elasticity form as much as possible.

(d) Is it possible that raising the tax could reduce the revenue the government receives? Prove or disprove by signing the derivative from (c).

(e) Explain in words the logic of part (d). Make sure you use the word "elasticity" in your answer.

4. *Educated Mothers*. A strong finding by development economists around the world is that when women are better educated, not only does their own standard of living rise, but so does their children's. The effect is largely due to the education of the mother herself, but also the average educational level of women in the community makes a difference. Suppose for example that a typical woman's utility function is $u_w(x, e) = x^{3/4}e^{1/4}$ where e is her own educational level and that a typical child's utility function is $u_c(e, E) = 12e^{1/16}E^{3/16}$ where e is the child's mother's education level and E is the average educational level of other women. Let there be one thousand women in the community. Note the child just takes x , e , and E as given.

- (a) Is there a positive externality in consumption of e ? How does it operate? Do you expect that this externality will be internalized in any way? Intuitively (no math), what is the difference between the free-market e and the socially optimal levels?
- (b) Suppose the price of x is $p_x = 1$ and the price of e is p_e . What is the woman's MRS in (x, e) space? If the woman's income is 2,000, what is her private demand curve for education?
- (c) Suppose a social planner cares equally about women and children and that each woman has exactly three children. What is a social planner's MRS in (x, e) space? If the the a typical woman's income is 2,000, what is the social demand curve for education? Graph the two curves. What Pigouvian subsidy would correct the externality?

Review Problems, not to turn in:

5. *Sopranos*. There are two goods, numeraire x and cooking c . The price of numeraire is always 1 throughout this problem, and the price of cooking is p_c .

Mrs. Soprano and Mrs. Bucco both have the same utility function:

$$u(x, c) = x^{0.8}c^{0.2}$$

Mrs. Soprano's endowment is $(\omega_{Sx}, \omega_{Sc}) = (100, 10)$. Mrs. Bucco's endowment is $(\omega_{Bx}, \omega_{Bc}) = (10, 10)$.

With this utility function and these endowments, the demand functions for numeraire for Mrs. Soprano and Mrs. Bucco are

$$x_S = 0.8 \frac{100 + 10p_c}{1} \quad x_B = 0.8 \frac{10 + 10p_c}{1}$$

- (a) If the two women can trade in an Edgeworth Box, what will be the final allocation and what will be the price of cooking?
- (b) Suppose that the "powers that be" decide that this final allocation is not all right. They want the final allocation to be $(x_B, c_B) = (66, 12)$. Note that $(66, 12)$ IS on the contract curve. What lump sum taxes and subsidies on the numeraire are necessary to make this happen? Illustrate with an Edgeworth Box diagram.

6. *Ethanol*. In the United States, the Federal Government gives a "blenders credit" of 5.1 cents per gallon of ethanol. This subsidy goes to oil refineries who buy the ethanol in order to blend into their gasoline. Thus, from the point of view of the *ethanol* market, the blenders credit is a per-unit subsidy to *consumers* of ethanol. To model this, let demand and supply be

$$x(p) = a - b(p - g) \quad s(p) = \alpha p$$

where a , ϵ , and α are parameters, p is price in cents per gallon, and g represents the blenders credit.

- (a) What is the equilibrium price and quantity?
- (b) Find the formula for how much the government pays out equilibrium? Use the derivative to discuss how the government payout changes as g changes? What is the *elasticity* of the government payout with respect to the subsidy?
- (c) Show social and private demand curves and the deadweight loss on a graph.

7. Levin. This question is inspired by Senator Carl Levin's report on gas prices (April 2002).

- (a) There is a rule of thumb in the oil industry that each 10 cent increase in the price of gas adds \$10 billion to oil industry revenues. This implies that

$$0.10 \frac{dTR}{dp} = 10,000,000,000$$

Show that you can obtain an elasticity estimate of $\epsilon = -0.23$ from this formula if you also know that the total quantity of gas consumed per year is 130 billion gallons.

- (b) The average American spends \$1,060 per year on gas and consumes 700 gallons. Let us suppose that the average American has an income of $m = 50,000$. Suppose you want to calibrate a demand curve of the following form :

$$y(p) = Amp^\epsilon$$

Show that $A = 0.0154$.

- (c) Perhaps we have chosen a bad demand function. Consider the following two demand functions:

$$y(p) = A\sqrt{m}p^\epsilon$$

$$y(p) = Am^2p^\epsilon$$

Draw the Engel curves that correspond to these functions.
Which one is more reasonable for gas?

8. *Slownet*. You opened an Internet Service Provider called Slow.net. You currently have 4,000 subscribers and 1,200 modems. No more than 25% of your customers are ever online at one time. You charge \$20 per month for a subscription. You hear that America Online has a price elasticity of -1.2 , and you think your own elasticity is the same.
- (a) Your marketing expert has suggested that lowering your price would raise total revenue. Use the derivative of total revenue to prove whether this is true or false.
 - (b) Suppose you lowered the price to \$15. How many subscribers would you expect to get, assuming you have a linear demand curve?
 - (c) Do you need more modems now that the price is \$15?
 - (d) Find your revenue at the price of \$20 and \$15. Explain the difference in light of part (a).
9. *Thornton*. Suppose the mayor of Middletown proposes a new tax on restaurant meals to finance Main Street improvements. Restaurant meals are elastically supplied at $s(p_s) = -5600 + 400p_s$. The tax will be a per unit tax, so the price restauranters receive is p_s and the price diners must pay is $p = p_s + t$. Demand for restaurant meals is $x(p) = 500 - 3p$.
- (a) Show the equilibrium price and quantity without the tax are \$15.14 and 456 respectively. Find the demand and supply elasticities at this equilibrium, and explain (in words) who will pay the tax, producers or consumers?

- (b) Show that the change in p_s when there is a change in the tax is 0.00744. Use the total derivative of the equilibrium condition.
- (c) Find a formula for $p_s(t)$, the equilibrium producer price given a tax of t . Then find formulas for $S(p_s(t))$, government revenue, and for deadweight loss as functions of t . The changes in government revenue and deadweight loss are respectively:

$$\frac{dR}{dt} = 456 - 5.96t \quad \frac{dDWL}{dt} = 2.98t$$

- (d) Is it possible for the mayor to get in a situation where he or she cannot raise enough tax revenue to fund the improvements without causing more deadweight loss than the gains to Middletown from having the improvements? Explain with reference to the above formulas and to the elasticities of supply and demand.

Answer to Review Problems:

5. *Sopranos_a*.

- (a) There are 110 units of numeraire in the economy, so we need

$$x_S + x_B = 0.8 \frac{100 + 10p_c}{1} + 0.8 \frac{10 + 10p_c}{1} = 110$$

Solving this gives $p_c = 1.375$.

This means that $x_S = 91$, $x_B = 19$, $c_S = 16.55$, $c_B = 3.45$. That is, Mrs. Bucco sells some cooking to Mrs. Soprano in exchange for numeraire.

- (b) Because of Walras' Law, all we need is to consider the demands for x . Note that if we take some amount of numeraire t from Mrs. Soprano and give it to Mrs. Bucco, the two women's

demand curves become

$$x_S = 0.8 \frac{100 - t + 10p_c}{1}$$

$$x_B = 0.8 \frac{10 + t + 10p_c}{1}$$

When we add these up and set equal to 110, the lump-sum transfer t just cancels out, so the price of cooking is still $p_c = 1.375$. Then all we have to do is make sure that Mrs. Bucco consumes $x_B = 66$ in the final allocation, and we're done. Thus:

$$x_B = 0.8 \frac{10 + t + 10 \cdot 1.375}{1} = 66$$

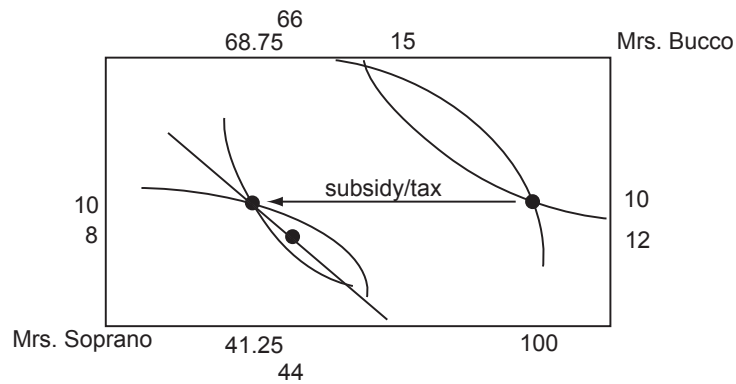
$$19 + 0.8t = 66$$

$$t = 58.75$$

To confirm this all works, consider that Mrs. Soprano must therefore consume the following amount of cooking:

$$c_S = 0.2 \frac{100 - 58.75 + 10 \cdot 1.375}{1.375} = 8$$

Since there are 20 units of cooking total and the goal was to have Mrs. Bucco consume 12 of them, Mrs. Soprano should consume 8, so this checks out. Note that the tax scheme reverses the trading: now Mrs. Soprano cooks for Mrs. Bucco!



6. *Ethanol_a*.

(a) Equilibrium p solves

$$\begin{aligned}a - b(p - g) &= \alpha p \\a + bg &= (\alpha + b)p \\p^* &= \frac{a + bg}{\alpha + b}\end{aligned}$$

So the equilibrium quantity is

$$s(p^*) = \alpha \frac{a + bg}{\alpha + b}$$

(b) The total government payout is $s(p^*)g$, which can be written:

$$R = g\alpha \frac{a + bg}{\alpha + b}$$

The change with respect to g is:

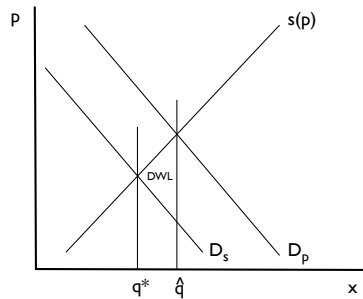
$$\frac{dR}{dg} = \alpha \frac{a + 2bg}{\alpha + b}$$

All of these terms are positive, so the government payout increases in g (not surprising).

The elasticity of the payout with respect to g is

$$\frac{\% \Delta R}{\% \Delta g} = \frac{dR}{dg} \frac{g}{R} = \alpha \frac{a + 2bg}{\alpha + b} \frac{\alpha + b}{\alpha(a + bg)} = \frac{a + 2bg}{a + bg}$$

(c) This subsidy is controversial because there is a disagreement over whether it corrects for a market failure or not. Assuming there is no underlying market failure, this is a standard case of a subsidy creating a distortion in the market: the social demand curve is simply the original curve, but the private demand curve is shifted by the subsidy. The deadweight loss, as always, is between correctly measured (social) demand curve and the correctly measured supply curve.



Note that you can make an argument that there *is* a market failure, in which case the labeling of the demand curves is reversed. This also changes the location of the deadweight loss (which is now avoided in this interpretation) because DWL is always between the correctly measured curves.

7. Levin_a.

- (a) We find this by expanding the derivative of total revenue and manipulating it to get the elasticity formula:

$$\begin{aligned}
 0.10 \frac{dTR}{dp} &= 10,000,000,000 \\
 0.10 \left(y + p \frac{dy}{dp} \right) &= 10,000,000,000 \\
 0.10y \left(1 + \frac{p}{y} \frac{dy}{dp} \right) &= 10,000,000,000 \\
 (1 + \epsilon) &= \frac{100,000,000,000}{y} \\
 \epsilon &= \frac{100,000,000,000}{130,000,000,000} - 1 \\
 \epsilon &= 0.77 - 1 \\
 \epsilon &= -0.23
 \end{aligned}$$

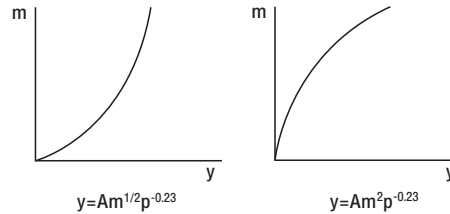
- (b) First, the average price of gas must be $\frac{1060}{700} = 1.51$. Given that, we need to fit the demand curve:

$$700 = A \cdot 50,000 \cdot 1.51^{-0.23}$$

$$0.014 = A \cdot 0.91$$

$$0.0154 = A$$

(c) The Engel curves depend on the m term only, and look like:



The first function is probably more reasonable, because we would expect gas to be a necessity, rising with income but not at as great a rate. Of course, up to a point, SUVs, ATVs, and boats might make gas a luxury.

8. *Slownet_a.*

(a)

$$R = px(p) \quad \frac{dR}{dp} = p \frac{dx}{dp} + x(p)$$

$$\frac{dR}{dp} < 0 \Rightarrow p \frac{dx}{dp} < -x(p)$$

$$\Rightarrow \frac{dx}{dp} \frac{p}{x(p)} < -1 \Rightarrow \epsilon < -1$$

Since $\epsilon = -1.2$ this condition holds, and we do predict that a decrease in price will increase revenue.

(b) If demand is $a - bp$, its slope is $-b$. If we plug $p = 20$, $x = 4000$ into the elasticity formula, we get

$$\frac{dx}{dp} \frac{p}{x(p)} = -b \frac{20}{4000} = -1.2 \Rightarrow b = 240$$

Then $a - 240(20) = 4000 \Rightarrow a = 8800$.

At $p = \$15$, $8800 - 240 \cdot 15 = 5200$.

- (c) $.25 \cdot 5200 = 1300 > 1200$, so yes, more modems will be needed.
- (d) $15 \cdot 5200 = \$78000$. $20 \cdot 4000 = \$80000$.

A linear demand curve does not have a constant elasticity. Therefore, although the elasticity may be -1.2 when the price is \$20, this does not hold for $p = \$15$. The elasticity is calculated at a point, just like a derivative. When the price moves far from this point (in this case it falls 25%), the approximation becomes inaccurate.

9. *Thornton_a.*

- (a) Setting demand equal to supply gives:

$$500 - 3p_s = -5600 + 400p_s \Rightarrow p_s = 15.14, \quad s(15.14) = 456$$

The elasticities are $\epsilon = -0.1$ and $\epsilon_s = 13.28$.

- (b) The equilibrium condition is

$$500 - 3(p_s + t) = -5600 + 400p_s$$

The total derivative is then

$$-3 \left(\frac{dp_s}{dt} + 1 \right) = 400 \frac{dp_s}{dt}$$

We can solve this for

$$\frac{dp_s}{dt} = -0.00744$$

- (c)

$$500 - 3(p_s + t) = -5600 + 400p_s$$

$$6100 - 3t = 403p_s$$

$$p_s = 15.14 - \frac{3}{403}t$$

$$S(p_s) = -5600 + 400\left(15.14 - \frac{3}{403}t\right) = 456 - 2.98t$$

$$\begin{aligned}\frac{dR}{dt} &= \frac{dS(p_s)}{dt} \\ &= \frac{d456t - 2.98t^2}{dt} = 456 - 5.96t\end{aligned}$$

$$\begin{aligned}\frac{dDWL}{dt} &= \frac{d\frac{1}{2}(456 - s(p_s))t}{dt} \\ &= \frac{d\frac{1}{2}(456 - 456 + 2.98t)t}{dt} \\ &= \frac{d1.49t^2}{dt} = 2.98t\end{aligned}$$

- (d) We know supply is very elastic and demand very inelastic. That means that adding a tax will basically increase the price to consumers a lot. As the tax increases, the marginal government revenue added goes down while the deadweight loss rises. Eventually, you reach a tax such that

$$\frac{dR}{dt} = 456 - 5.96t = 2.98t = \frac{dDWL}{dt} \Rightarrow t = 51$$

So, once the tax reaches 51, each marginal increase in tax causes more DWL than it does tax revenue.