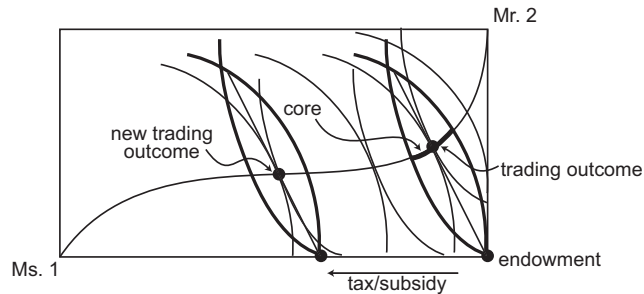


ECON 301, Professor Hogendorn

Problem Set 4 Answers

1. RichAndPoor_a.

- (a) To show the relatively low value both people put on good y , we need a large MRS (steeply sloped indifference curves).



- (b) Although the center point would then be feasible from the point of view of the budget line, it would not be a Walrasian equilibrium. The indifference curves of the two consumers would be tangent at two different points along this budget line, so there would not be a market-clearing equilibrium. Some gains from trade would be lost, and the center point would not actually be achieved. In any case, unless the center point lies exactly on the contract curve, it is not efficient, since the consumers can Pareto-improve on it.
- (c) The second welfare theorem says that any point on the contract curve can be supported as a Walrasian equilibrium provided the consumers begin at the proper endowment point. In the graph, the government could lump-sum-redistribute good x from Mr. 2 to Ms. 1. Then the agents can trade at free-market prices and come as close to the center of the box as the contract curve will allow.

2. *CokePepsi_a.*

- (a) Both have positive income elasticity, so both are normal goods. Coke income elasticity is less than 1, so it is a necessity, while Pepsi is a luxury with an income elasticity greater than 1.
- (b) Using the formula for income elasticity, we have

$$\epsilon_m^{\text{coke}} = \frac{\partial q_{\text{coke}}}{\partial m} \frac{m}{q_{\text{coke}}} = \frac{\partial q_{\text{coke}}}{\partial m} \frac{m}{1} = 0.58$$

We can solve this for

$$\frac{\partial q_{\text{coke}}}{\partial m} =$$

Doing the same for Pepsi gives

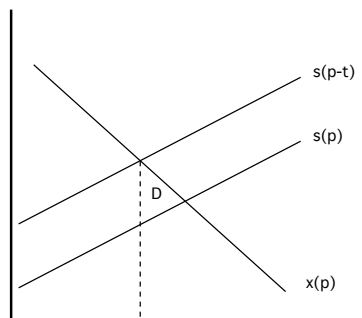
$$\frac{\partial q_{\text{pepsi}}}{\partial m} =$$

- (c) It makes sense to think that Coke and Pepsi are substitutes for each other. That would mean that if the price of one went up, people would buy more of the other, so the signs should be positive.
- (d) The differential is

$$dq_{\text{coke}} = \frac{\partial q_{\text{coke}}}{\partial p_{\text{coke}}} dp_{\text{coke}} + \frac{\partial q_{\text{coke}}}{\partial p_{\text{pepsi}}} dp_{\text{pepsi}} + \frac{\partial q_{\text{coke}}}{\partial m} dm$$

3. *Laffer_a.*

- (a) The graph is:



The DWL, labeled D, is the consumer plus producer surplus that is lost when the tax reduces the quantity consumed below the original equilibrium.

(b) We know that supply must equal demand always:

$$x(p) = s(p - t)$$

Now we totally differentiate and rearrange the above:

$$\frac{\partial x}{\partial p} dp = \frac{\partial s}{\partial p} dp - \frac{\partial s}{\partial p} dt$$

(Note that the function $s(\cdot)$ only has one argument, therefore whether the p or the t in that one argument changes, the relevant change in s can still be denoted $\partial s / \partial p$.)

Now we solve for dp/dt :

$$\begin{aligned} \left(\frac{\partial x}{\partial p} - \frac{\partial s}{\partial p} \right) dp &= - \frac{\partial s}{\partial p} dt \\ \frac{dp}{dt} &= - \frac{\frac{\partial s}{\partial p}}{\frac{\partial x}{\partial p} - \frac{\partial s}{\partial p}} \end{aligned}$$

Now multiply the right hand side by $(p/x)/(p/x)$ where x is the equilibrium quantity and is also equal to s :

$$\frac{dp}{dt} = - \frac{\epsilon_S}{\epsilon_D - \epsilon_S} = \frac{\epsilon_S}{\epsilon_S - \epsilon_D}$$

(c)

$$\begin{aligned} TR &= tx(p) \\ \frac{dTR}{dt} &= t \frac{\partial x}{\partial p} \frac{dp}{dt} + x(p) \\ &= t \frac{\partial x}{\partial p} \frac{dp}{dt} + x(p) \\ &= t \frac{\partial x}{\partial p} \frac{\epsilon_S}{\epsilon_S - \epsilon_D} + x(p) \end{aligned}$$

$$\begin{aligned}
&= x(p) \left(t \frac{\partial x}{\partial p} \frac{1}{x(p)} \frac{\epsilon_S}{\epsilon_S - \epsilon_D} + 1 \right) \\
&= \frac{x(p)}{p} \left(t \frac{\epsilon_D \epsilon_S}{\epsilon_S - \epsilon_D} + p \right)
\end{aligned}$$

(d) For $\frac{dTR}{dt} < 0$, we can simplify the above further:

$$\begin{aligned}
\frac{dTR}{dt} &< 0 \\
\frac{x(p)}{p} \left(t \frac{\epsilon_D \epsilon_S}{\epsilon_S - \epsilon_D} + p \right) &< 0 \\
t \frac{\epsilon_D \epsilon_S}{\epsilon_S - \epsilon_D} + p &< 0 \\
\frac{\epsilon_D \epsilon_S}{\epsilon_S - \epsilon_D} &< -\frac{p}{t} \\
-\frac{t}{p} &< \frac{\epsilon_S - \epsilon_D}{\epsilon_D \epsilon_S} \\
\frac{t}{p} &> -\left(\frac{1}{\epsilon_D} - \frac{1}{\epsilon_S} \right) \\
\frac{t}{p} &> \frac{1}{|\epsilon_D|} + \frac{1}{\epsilon_S}
\end{aligned}$$

Sure, this could happen. You would need demand and supply to be relatively elastic and the tax rate t/p to be relatively high.

4. EducatedMothers_a.

(a) Yes, there is a positive externality. The mother chooses the amount of education, but her children benefit directly and all other children benefit to the extent that the average level of education rises.

The externality may or may not be internalized depending on how you interpret the problem. The hard-nosed economist would answer that any concern the woman has for her child should already be reflected in her utility function. After all,

her utility function is supposed to be a complete measure of her happiness -- if she cares about her child, then that should be in there.

In any case, it seems very unlikely that the mother would take account of the small effect that her own education has on the average level of education, so a positive externality should persist. Intuitively, the mother takes account of her own utility only, while a social planner would also take account of the effect on the average level. Thus, the socially optimal e would be higher than the free-market e .

(b) Using the hard-nosed logic from (a), the woman's MRS is:

$$MRS = -\frac{MU_x}{MU_e} = -\frac{\frac{3}{4}x^{-1/4}e^{1/4}}{\frac{1}{4}x^{3/4}e^{-3/4}} = -3\frac{e}{x}$$

To find her demand curve, we set the MRS equal to the ratio of the prices and then substitute into the demand function:

$$-3\frac{e}{x} = -\frac{1}{p_e} \Rightarrow 3p_e e = x$$

$$p_e e + x = 2000 \Rightarrow 4p_e e = 2000 \Rightarrow e(p_e) = \frac{2000}{4p_e}$$

(c) The social planner's utility includes the 1000 mothers and their 3000 children:

$$u_s(e, x) = 1000x^{3/4}e^{1/4} + 3000 \cdot 12e^{1/16}e^{3/16}$$

Note the way the big E is just replaced by little e for the social planner since the planner will decide every woman's e and thus determine the average at the same time. The social planner's MRS is:

$$MRS = -\frac{MU_x}{MU_e} = -\frac{1000\frac{3}{4}x^{-1/4}e^{1/4}}{1000\frac{1}{4}x^{3/4}e^{-3/4} + 36,000\frac{1}{4}e^{-3/4}}$$

Since there is more “stuff” in the denominator for the social planner, the social planner will always have a smaller (in absolute value) MRS than the mother on her own.

Now set the social MRS equal to the slope of the budget line ($-p_x/p_e = -1/p_e$):

$$-\frac{1000\frac{3}{4}x^{-1/4}e^{1/4}}{1000\frac{1}{4}x^{3/4}e^{-3/4}+36,000\frac{1}{4}e^{-3/4}} = -\frac{1}{p_e}$$

$$p_e 1000\frac{3}{4}x^{-1/4}e^{1/4} = 1000\frac{1}{4}x^{3/4}e^{-3/4} + 36,000\frac{1}{4}e^{-3/4}$$

$$p_e = \frac{1}{3}\frac{x}{e} + 12\frac{x^{0.25}}{e}$$

$$p_e e = \frac{1}{3}x + 12x^{0.25}$$

And substitute into the budget line:

$$p_e e + x = 2000 \Rightarrow \frac{1}{3}x + 12x^{0.25} + x = 2000 \Rightarrow \frac{4}{3}x + 12x^{0.25} = 2000$$

This is slightly tricky to solve. Let $y = x^{0.25}$. Then we can rewrite the equation as:

$$\frac{4}{3}y^4 + 12y = 2000$$

Using the polynomial route finder on a calculator, the only positive real route is $y = 6.16$, which implies that $x = 1445$. Then using the formula for $p_e e$, we can find the social demand curve for education:

$$p_e e = \frac{1}{3}1445 + 12(1445^{0.25}) \Rightarrow e^{\text{soc}}(p_e) = \frac{555.6}{p_e}$$

This is similar to the private demand curve, but with a larger numerator. It is clear that the social demand curve will lie above and to the right of the private demand curve. The Pigou-vian subsidy that shifts the private demand curve up to match the social demand curve solves:

$$e((1-s)p_e) = e^{\text{soc}}(p_e) \Rightarrow \frac{500}{(1-s)p_e} = \frac{555.6}{p_e} \Rightarrow s = 10\%$$