ECON 301, Professor Hogendorn

Problem Set 5

1. *Obesity.* This problem asks you to analyze the obesity epidemic using methods developed in Dupor and Liu, "Jealousy and Equilibrium Overconsumption," *American Economic Review,* March 2003.

   (a) Suppose that utility for food consumed (c), the typical American's food consumption (x), and labor (n) is given by

   $$U(c,x,n) = \left(\frac{c^{1/2} - \frac{1}{2}x^{1/2}}{\frac{1}{2}}\right)^{\frac{3}{2}} - n$$

   Assume that $c > 0.25x$. If ``jealousy'' is defined as $U_2 < 0$, show whether this utility function exhibits jealousy.

   (b) Suppose that the labor needed to buy food is given by $c = wn$ where w is some parameter representing the wage. Set up the consumer's utility maximization choice of c using the method of substitution. What is the first order condition?

   (c) What is the marginal rate of substitution in (n, c) space? (Note, this is extremely easy once you've done part (b).) If the wage rises (w goes up), how does this change the first order condition and the MRS? What happens to consumers' food consumption?

   (d) Write down the maximization problem and first order condition for a social planner. At the symmetric solution to (b), where $c = x$, does the planner give people as much food to consume as they would choose by themselves? Why or why not?

2. *Sigma.* Consider the production function

$$f(L, K) = (L^{0.25} + K^{0.25})^3$$

(a) What is the formula for MRTS?

(b) Does this production function exhibit decreasing, constant, or increasing returns to scale?

(c) What is the elasticity of substitution?

Review Problems, not to turn in:

3. Do problem 11.6 parts a and b from the Nicholson reading.

4. Do problem 11.1 from the Nicholson reading.

Answer to Review Problems:

4. *Nicholson11.6_a.*

(a)

$$\begin{aligned} \frac{\partial q}{\partial K} &= \frac{1}{\rho}(K^\rho + L^\rho)^{\frac{1}{\rho}-1}\rho K^{\rho-1} \\ &= (K^\rho + L^\rho)^{\left(\frac{1}{\rho}\right)^{1-\rho}} K^{\rho-1} \\ &= \frac{q^{1-\rho}}{K^{1-\rho}} \end{aligned}$$

The same technique works for L.

(b) The MRTS is:

$$\text{MRTS} = -\frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial K}} = -\frac{\frac{1}{\rho}(L^\rho + K^\rho)^{\frac{1}{\rho}-1}\rho L^{\rho-1}}{\frac{1}{\rho}(L^\rho + K^\rho)^{\frac{1}{\rho}-1}\rho K^{\rho-1}} = -\frac{L^{\rho-1}}{K^{\rho-1}} = -\frac{K^{1-\rho}}{L^{1-\rho}}$$

(I believe Nicholson made a typo here, reversing L and K in his book -- see his footnote 25 for the correct MRTS.) To find the elasticity of substitution solve:

$$\begin{aligned}
\frac{\partial\, ln\, K/L}{\partial\, ln\, |MRTS|} &= \frac{\partial\, ln\, K/L}{\partial\, ln\, \frac{K^{1-\rho}}{L^{1-\rho}}} \\
&= \frac{\partial\, ln\, K/L}{\partial (1-\rho)\, ln\, K/L} \\
&= \frac{1}{1-\rho}
\end{aligned}$$

5. *Nicholson11.1_a*

   (a) The graph is upward sloping and concave.

   (b) $AP_L = \frac{100\sqrt{L}}{L} = 100L^{-0.5}$. This is clearly decreasing in L.

   (c) $\frac{dq}{dL} = 100 \cdot 0.5L^{-0.5} = \frac{50}{\sqrt{L}}$. Because the total product of L is concave everywhere, it is always the case that MP_L is diminishing. That means that each additional worker always pulls down the average product.