ECON 301, Professor Hogendorn

Problem Set 5 Answers

1. *Obesity_a.*

   (a) We want to find the derivative with respect to x:

$$\begin{aligned} U_2 &= -\frac{3}{2}\left(\frac{c^{1/2}-\frac{1}{2}x^{1/2}}{\frac{1}{2}}\right)^{1/2}\frac{1}{2}x^{-1/2} \\ &= -\frac{3}{4}\left(\frac{c^{1/2}-\frac{1}{2}x^{1/2}}{\frac{1}{2}x}\right)^{1/2} \end{aligned}$$

   If $c > 0.25x$, then $c^{1/2} > 0.5x^{1/2}$ and the numerator on the fraction is positive. The entire expression must be negative.

   (b) The first step is to figure out how much labor the person needs to supply. It is $c = wn \Rightarrow n = \frac{c}{w}$. This allows the utility max problem to be set up by substitution:

$$\max_c U(c, x, c/w) = \left(\frac{c^{1/2}-\frac{1}{2}x^{1/2}}{\frac{1}{2}}\right)^{3/2} - \frac{c}{w}$$

   Then the first order condition is:

$$U_1 + \frac{1}{w}U_3 = \frac{3}{2}\left(\frac{c^{1/2}-\frac{1}{2}x^{1/2}}{\frac{1}{2}c}\right)^{1/2} - \frac{1}{w} = 0$$

   (c) The MRS can be found simply by appealing to its definition:

$$MRS = -\frac{\frac{\partial U}{\partial n}}{\frac{\partial U}{\partial c}} = -\frac{-1}{\frac{3}{2}\left(\frac{c^{1/2}-\frac{1}{2}x^{1/2}}{\frac{1}{2}c}\right)^{1/2}} = \frac{2}{3}\left(\frac{\frac{1}{2}c}{c^{1/2}-\frac{1}{2}x^{1/2}}\right)^{1/2}$$

   If the wage rises, the first order condition shifts up and to the right. Consumption must rise to preserve the equality. Since c rises, the MRS will rise.

(d) The planner will choose everyone's c all at once, thus solving:

$$\max_c U(c, c, c/w) = \left(\frac{c^{1/2} - \frac{1}{2}c^{1/2}}{\frac{1}{2}} \right)^{3/2} - \frac{c}{w} = c^{3/4} - \frac{c}{w}$$

The planner's first order condition has an extra term:

$$U_1 + U_2 + \frac{1}{w}U_3 = 0$$

In this case it is much simpler just to take the derivative of the simplified objective function:

$$\frac{3}{4}c^{-1/4} - \frac{1}{w} = 0$$

but the question can also be answered just by noting that U_2 is less than 0 because of jealousy, so the planner's FOC must be smaller than the private FOC. As we saw in part (c), to make the FOC smaller means making c smaller. The planner gives less food to the consumers because the planner takes into account the disutility that results from "jealousy."

2. *Sigma_a.*

(a) The MRTS is:

$$\begin{aligned} MRTS &= -\frac{3(K^{0.25} + L^{0.25})^2 0.25 L^{-0.75}}{3(K^{0.25} + L^{0.25})^2 0.25 K^{-0.75}} \\ &= \frac{L^{-0.75}}{K^{-0.75}} \end{aligned}$$

(b) There are decreasing returns to scale:

$$f(2K, 2L) = ((2K)^{0.25} + (2L)^{0.25})^3 = 1.68(K^{0.25} + L^{0.25})^3 < 2f(K, L)$$

(c) Recall that $\sigma = \frac{d \ln K/L}{d \ln MRTS}$. In this case that is:

$$\frac{d \ln K/L}{d 0.75 \ln K/L} = 1.33$$

This is an example of a constant elasticity of substitution (CES) production function, and as the name suggests we found that σ is a constant.