ECON 301, Professor Hogendorn

Problem Set 6

1. *Tequila.* The spirit tequila is produced by distilling the fermented juice of the agave plant. True tequila can only be made from agave grown in the officially denominated tequila region in the environs of Tequila, Mexico. An agave plant takes 8 years to reach maturity, and the region was hit by a freak frost in 1997 that killed many agave plants.

   Suppose that tequila is produced according to the following production function:

   $$f(x, a) = 143x^{0.05}a$$

   a is metric tons of agave and x is a composite factor including labor, oak casks, grinding equipment, and so forth. The idea behind this production function is that if $x = 1$, each ton of agave produces 143 liters of tequila, but x could be adjusted to change this amount.

   Let the price of X be 400 pesos and the price of a metric ton of agave is 1,000 pesos.

   (a) What is the long-run cost curve for tequila?

   (b) If distillers set LRAC(q)=6, how much a do they use?

   (c) Suppose that after the freeze, 90% of the amount of a from part (b) remains available, and so it becomes a fixed factor. If, nevertheless, distillers want to produce the same ouput, what is the cost?

2. *LRSR*. A firm has the following production function:

$$f(K, L) = \sqrt{K} + \sqrt{L}$$

where K is capital and L is labor. The price of K is w_K and the price of L is w_L.

(a) Use the Lagrangian to confirm that the long-run conditional factor demand functions for K and L are

$$K(w_K, w_L, y) = \frac{y^2}{\left(1 + \frac{w_K}{w_L}\right)^2}$$

$$L(w_L, w_K, y) = \frac{y^2}{\left(1 + \frac{w_L}{w_K}\right)^2}$$

(b) If $w_K = 1$, $y = 90$, and $w_L = 2$ what is the LRMC?

(c) Suppose that $y = 90$ happens to be the output at which LRAC is equal to SRAC. Assume capital is fixed in the short run and labor is variable, and continue to assume that $w_K = 1$ and $w_L = 2$. What is SRMC when $y = 120$?

(d) Draw a graph of the LRAC, LRMC, SRAC, SRMC.

Review problems only, not to turn in:

3. *VisaDiscover*. Visa and Discover are considering the introduction of debit cards. Both firms have the same production function $f(L, K) = L^{.8}K^{.3}$. Labor and capital both cost \$10 per unit.

(a) What is the long run total cost curve for either company? Use the Lagrangian to show your answer.

(b) Assume K is fixed in the short run. Confirm that the short-run total cost curve is $TC(y|K) = 10K + 10K^{-0.375}y^{1.25}$.

Answers to Review Problems:

3. *VisaDiscover_a.*

(a)

$$\begin{aligned}
\max_{K,L,\lambda} \mathcal{L} &= 10K + 10L - \lambda(K^{0.3}L^{0.8} - y) \\
\frac{\partial \mathcal{L}}{\partial K} &= 10 - \lambda 0.3K^{-0.7}L^{0.8} = 0 \\
\frac{\partial \mathcal{L}}{\partial L} &= 10 - \lambda 0.8K^{0.3}L^{-0.2} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= K^{0.3}L^{0.8} - y = 0
\end{aligned}$$

Solving simultaneously we get:

$$\begin{aligned}
\lambda = 33.33K^{0.7}L^{-0.8} & \\
\lambda = 12.5K^{-0.3}L^{0.2} & \qquad K = 0.375L \\
(0.375L)^{0.3}L^{0.8} - y = 0 & \Rightarrow \quad 0.75L^{1.1} - y = 0 \\
L^* = 1.3y^{0.91} & \qquad K^* = 0.5y^{0.91}
\end{aligned}$$

$$TC(y) = 10K^* + 10L^* = 13y^{0.91} + 5y^{0.91} = 18y^{0.91}$$

(b)

$$y = K^{0.3}L^{0.8} \Rightarrow L^{0.8} = K^{-0.3}y \Rightarrow L(y|K) = K^{-0.375}y^{1.25}$$

$$TC(y|K) = 10K + 10L(y|K) = 10K + 10K^{-0.375}y^{1.25}$$