ECON 301, Professor Hogendorn

Problem Set 6 Answers

1. *Tequila_a.*

   (a) Let's use the MRTS to find the answer:

$$\text{MRTS} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial a}} = -\frac{0.05 \cdot 143ax^{-0.095}}{143x^{0.05}} = -0.05\frac{a}{x}$$
$$-0.05\frac{a}{x} = -\frac{400}{1000} \Rightarrow \frac{a}{x} = 8 \Rightarrow a = 8x$$

   Then $q = 143(8x) \cdot x^{0.05} = 1144x^{1.05}$ and inverting this we find that to produce q, the conditional factor demand is

$$x(q) = 0.0012q^{0.95}$$

   Thus the cost of q is:

$$TC(q) = 400 \cdot 0.0012q^{0.95} + 1000 \cdot 8 \cdot 0.0012q^{0.95} = 10.27q^{0.95}$$

   (b) The long run average cost is

$$AC(q) = \frac{TC(q)}{q} = 10.27q^{-0.05}$$

   If $AC(q) = 6$, then

$$10.27q^{-0.05} = 6 \Rightarrow q = 46433$$

   Now we use the conditional factor demand:

$$x(46433) = 32.56 \qquad a(46433) = 260.5$$

(c) The short-run problem is:

$$f(0.9 \cdot 260.5, x) = 143 \cdot 234.45 \cdot x^{0.05}$$
$$\Rightarrow q = 33526.35x^{0.05}$$

To produce output 46433, the producers need:

$$46433 = 33526.35x^{0.05} \Rightarrow x = 674.23$$

Thus, the new total cost is

$$TC(46433|a = 234.45) = 1000 \cdot 234.45 + 400 \cdot 674.23 = 504142.82$$

Thus, the freeze caused a huge cost increase, assuming production did not change.

2. *LRSR_a.*

(a) The Lagrangian is:

$$\begin{aligned}
\min_{K,L,\lambda} \mathcal{L} &= w_K K + w_L L - \lambda(\sqrt{K} + \sqrt{L} - y) \\
\frac{\partial \mathcal{L}}{\partial K} &= w_K - \lambda \frac{1}{2} \frac{1}{\sqrt{K}} = 0 \\
\frac{\partial \mathcal{L}}{\partial L} &= w_L - \lambda \frac{1}{2} \frac{1}{\sqrt{L}} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= \sqrt{K} + \sqrt{L} - y = 0
\end{aligned}$$

Solving simultaneously we get:

$$\lambda = 2w_K\sqrt{K}$$
$$\lambda = 2w_L\sqrt{L} \qquad \lambda = \lambda \Rightarrow 2w_K\sqrt{K} = 2w_L\sqrt{L}$$
$$\Rightarrow K = \frac{w_L^2 L}{w_K^2}$$

$$\sqrt{\frac{w_L^2 L}{w_K^2}} + \sqrt{L} - y = 0 \quad \Rightarrow \quad L(w_L, w_K, y) = \frac{y^2}{\left(1 + \frac{w_L}{w_K}\right)^2}$$
$$K(w_K, w_L, y) = \frac{y^2}{\left(1 + \frac{w_K}{w_L}\right)^2}$$

(b)

$$TC(y) = 1 \cdot (1, 2, y) + 2 \cdot L(2, 1, y)$$

$$TC(y) = \frac{y^2}{2.25} + 2\frac{y^2}{9}$$

$$LMC(y) = 2\frac{y}{2.25} + 4\frac{y}{9}$$

$$LMC(90) = 80 + 40 = 120$$

(c) $K(1, 2, 90) = 3600$ is the amount of capital available. Then the short run production function is:

$$f(3600, L) = 60 + \sqrt{L} \Rightarrow L = (y - 60)^2$$

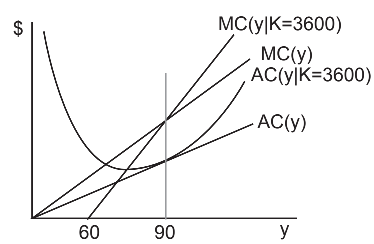
The short run total cost curve is:

$$TC(y|K = 3600) = 3600 + 2(y - 60)^2$$

The short run marginal cost curve is:

$$MC(y|K = 3600) = 4(y - 60) \quad \text{so} \quad MC(120|K = 3600) = 240$$

(d) A generic graph would look like Figure 21.10 of Varian. (But note that Varian's publisher is imprecise because the MC curves do not quite cut the AC curves at their lowest points.) Specifically for these cost functions, the graph is:



Note that when a firm is operating on the upward-sloping portion of LAC, the marginal cost curves are above the average cost curves.