ECON 301, Professor Hogendorn

Problem Set 7

1. *OilProducers.* Suppose there are two oil-producing regions in the world, and in each one there are perfectly competitive producers. The factor price for extraction equipment is w, and the long-run average costs of one of these firms are

$$AC(y) = \sqrt{w}(1 + y^2)$$

There are 20 firms in the Middle East, and for them extraction equipment costs 400. There are 10 firms in Alberta who must pay 900 for extraction equipment.

   (a) Suppose the world demands oil perfectly inelastically in amount $\hat{y}$. What value of $\hat{y}$ would result in a price of oil of \$25? (Be careful to think about how many firms produce oil at this level.)

   (b) What is the short-run market supply curve for oil (holding the number of firms fixed)?

   (c) What value of $\hat{y}$ would result in a price of oil of \$32?

2. *OilRefineries.* A common argument against environmental regulations is that they will act like a tax and raise the price of goods. When there is a lumpiness to costs, however, this may turn out not to be true. For example, in 1990 several amendments were passed to the Clean Air Act which required oil refineries to significantly upgrade their capital. The surprising result was that the refining industry increased its total supply and the market prices of refined products actually fell, *ceteris paribus*. Here's how this can happen.

Suppose the demand for oil in the U.S. is 15.4 million barrels per day, and is perfectly inelastic. Suppose we can treat all U.S. oil refineries as a single firm with production function

$$f(L, K) = 0.147L^{0.3}K^{0.6}$$

Let $w = 10$, but keep r as a parameter.

(a) Use the Lagrangian to find the conditional factor demands for labor and capital.

(b) Suppose that in 1989, the refining industry had $K = 359$ and was stuck in the short-run. Show that the short-run marginal cost curve for the whole industry is

$$MC(y|K = 359) = 0.153y^{2.33}$$

(c) During the early 1990's, oil refineries did not move into the long run because there were additional costs associated with shutting down the refineries in order to replace the capital. If the firms remain stuck in the short run, and if $r = 1$ what would be the price of oil assuming perfectly competitive behavior?

(d) Now suppose that the government regulation raises the cost of capital to $r = 1.15$ and it forces the firms to move to the long run so they can adjust their capital. If demand is still 15.4, what would be the price of oil?

(e) Graph the supply and demand equilibrium of parts (c) and (d).

(f) For each part above, a-d, what, in your opinion, is the most limiting assumption underlying the choice of modeling approach. For example for part (a), your answer might be "It is not realistic to treat all refineries as a single firm because." (This example is not a very good answer, however.) For each

limitation, say what is being being oversimplified, and how addressing the concern might be expected to change the answer.

3. *Coke.* Suppose that all around the world, there are small towns in which the price elasticity of demand for Coca-cola is constant at -1.2. Each of these towns is served by a monopoly Coke distributor. However, the technology for distributing Coke varies widely: huge bottling plants and 18-wheeler truck delivery in the USA, local bottlers and van delivery in Japan, delivery by pack mule to isolated parts of Bolivia, etc.

   (a) What is the Lerner Index on Coke in these markets?

   (b) Let the production function be $f(K) = \beta K^2$, where β varies from place to place, and let the price of capital be 20. How does the price of Coke vary with β? (This is pretty tricky. Note that there is a constant elasticity demand, check review problem *Minus2.*)

   Review problems only, not to turn in:

4. *Technologies.* Suppose there are three technologies for providing a new Internet service, and one of them will eventually emerge as clearly superior to the other two. All three technologies use the factors S for servers and B for bandwidth, and the factor prices are $w_S = w_B = 1$.

   Technology A has production function $F(S, B) = S^{0.7}B^{0.7}$ but S must be set to 10 units and can never be changed.

   Technology B has production function $F(S, B) = S^{0.6}B^{0.6}$ and both factors can be freely varied.

   Technology C has production function $F(S, B) = S^{0.3}B^{0.6}$ and both factors can be freely varied.

Which technologies have economies of scale? Which have diseconomies of scale?

5. *Consulting.* Technology can improve labor productivity. One might be concerned that this could be bad for workers since fewer would be needed to produce the same output. Displaced workers might have to move to another industry. To think about this, suppose an industry has the production function $f(L) = \alpha L^{0.5}$. The conditional factor demand is thus $L(y) = \left(\frac{1}{\alpha}\right)^2 y^2$. Let $w_L = 1$ throughout this whole problem (i.e. overall labor market equilibrium is unaffected by the changes in the industry we examine here). Suppose there is a fixed cost to start a firm which is $F = 2500$. The cost function is thus

$$c(y) = \left(\frac{1}{\alpha}\right)^2 y^2 + 2500$$

Note that this is both the short-run and the long-run cost function; the only difference is that in the long run a firm can exit or enter the industry.

Suppose that demand in the industry is given by $X(p) = 60000p^\epsilon$. Elasticity ϵ can take on two values: -0.5 and -1.5. Answer the following for each of these values:

(a) Suppose that initially $\alpha = 100$ and the industry is in long-run perfectly competitive equilibrium. How many firms are there? What is the total number of workers?

(b) Suppose that improved technology causes a change to $\alpha = 160$. In the short run (i.e. with the number of firms fixed) what is the new total number of workers?

(c) In the long run, the number of firms will adjust to the new situation. What is the new number of firms and the new total number of workers?

4

(d) Describe in words what an individual worker would experience during parts (a)-(c). For example, your description might read ``First I noticed that my firm hired a few new people. Later, some of our competitors went out of business. Most of the people who worked for those firms came to work at my firm and our remaining competitors, but a few had to get jobs in another industry.'' Remember to do this for both values of elasticity, and discuss which elasticity is preferable for the workers.

6. *Minus2.* Suppose the demand curve for a good is:

$$x(p) = 1000p^{-2}$$

There is a monopoly which produces this good, and it has constant marginal cost of $2 per unit.

(a) What is the monopoly optimal price, quantity, and profit?

(b) What is the deadweight loss of this monopoly?

Answers to Review Problems:

4. *Technologies_a.* Technology B always has economies of scale at all output levels because the exponents add to more than 1.

Technology C has exponents that add to less than 1, so there are diseconomies of scale at all levels of output.

For Technology A, we can rewrite the production function as $F(B) = 5B^{0.7}$. Thus, $B = 0.1y^{10/7}$ and $c(y) = 10 + 0.1y^{10/7}$. The average cost is $AC(y) = \frac{10}{y} + 0.1y^{3/7}$. The marginal cost is $MC(y) = \frac{1}{7}y^{3/7}$. To find the bottom of the U of average cost

$$
\begin{aligned}
AC(y) &= MC(y) \\
\frac{10}{y} + 0.1y^{3/7} &= \frac{1}{7}y^{3/7}
\end{aligned}
$$

$$\frac{10}{y} = 0.04y^{3/7}$$
$$10 = 0.043y^{10/7}$$
$$y = 45.34$$

So for y < 45.34 there are economies of scale, but for greater y there are diseconomies.

5. *Consulting_a.*

   (a) The average and marginal cost curves in this case are:

   $$AC(y) = \frac{2500}{y} + 0.0001y \qquad MC(y) = 0.0002y$$

   Thus, the minimum average cost is $\frac{2500}{y} + 0.0001y = 0.0002y \Rightarrow$ $y_{LR} = 5000$. At this output, the amount of labor employed by each firm is $L(5000) = 2500$.

   The marginal cost of this output level is $MC(5000) = 1$, and since perfectly competitive firms set price equal to marginal cost, we have $p_{LR} = 1$. This is the long run supply curve.

   Equating supply to demand, we find the demand at $p = 1$, which is $60000 \cdot 1^{\epsilon} = 60000$.

   The number of firms in the market must therefore be $N = \frac{60000}{5000} = 12$. Since each firm employs 2500 workers, total employment is 30000.

   (b) Now that $\alpha = 160$, the conditional factor demand is $L(y) = 0.00004y^2$ and the total cost function is $c(y) = 0.00004y^2 + 2500$. Thus, the new marginal cost curve and the short-run firm supply curve is:

   $$MC(y) = 0.00008y \qquad s(p) = 12500p$$

Since the number of firms cannot change in the short run, there are still 12 of them, so the market supply curve is just $12s(p)$, and setting supply equal to demand gives us:

$$
\begin{aligned}
60000p^\epsilon &= 12 \cdot 12500p \\
p^{\epsilon-1} &= 2.5 \\
p &= 2.5^{\frac{1}{\epsilon-1}}
\end{aligned}
$$

$$
\begin{aligned}
p &= (0.54, 0.69) \text{ when } \epsilon = (-0.5, -1.5) \\
X(p) &= (81650, 104683) \text{ when } \epsilon = (-0.5, -1.5) \\
y &= (6804, 8724) \text{ when } \epsilon = (-0.5, -1.5) \\
L(y) &= (1852, 3044) \text{ when } \epsilon = (-0.5, -1.5) \\
NL(y) &= (22224, 36528) \text{ when } \epsilon = (-0.5, -1.5)
\end{aligned}
$$

(c) With $\alpha = 160$, the average and marginal cost curves are:

$$
AC(y) = \frac{2500}{y} + 0.00004y \qquad\qquad MC(y) = 0.00008y
$$

Thus, the minimum average cost is $\frac{2500}{y}+0.00004y = 0.00008y \Rightarrow$ $y_{LR} = 7906$. At this output, the amount of labor employed by each firm is $L(7906) = 2500$. (Note this is the same as before, which occurs because we have only changed the coefficient on the production function.)

The marginal cost of this output level is $MC(7906) = 0.632$, and since perfectly competitive firms set price equal to marginal cost, we have $p_{LR} = 0.632$. This is the long run supply curve. Equating supply to demand, we find:

$$
\begin{aligned}
p &= (0.632, 0.632) \text{ when } \epsilon = (-0.5, -1.5) \\
X(p) &= (75473, 119420) \text{ when } \epsilon = (-0.5, -1.5) \\
N &= (9.54, 15.1) \text{ when } \epsilon = (-0.5, -1.5) \\
L(y) &= (2500, 2500) \text{ when } \epsilon = (-0.5, -1.5) \\
NL(y) &= (23866, 37750) \text{ when } \epsilon = (-0.5, -1.5)
\end{aligned}
$$

(d) Case of $\epsilon = -0.5$: I never should have taken the job at Sprint. Everything was fine until stupid researchers at Bell Labs and Nortel introduced the new technology. There was overcapacity everywhere, and Sprint laid off about 25% of its workforce. After a while, Global Crossing, Williams, and Worldcom filed for bankruptcy. But now that there's been some consolidation, Sprint is doing a little better, and it looks like the laid-off workers will be rehired. My friends at Global Crossing are out of luck though -- there won't be any telecoms jobs for them.

Case of $\epsilon = -1.5$: When I started at Nortel, it seemed like a sleepy firm, but then this great new technology came along. Nortel grew really fast, and we hired all kinds of new people. Fortunately, I saw that the good times couldn't last, so I cashed in my stock options and moved to a startup. It's a good thing, because Nortel laid off most of the people it hired. My new firm's hanging in there, but it's not like the boom times.

Clearly, the $\epsilon = -1.5$ is preferable, but note that even then there were some layoffs in this model. Also note that both examples provide a somewhat reasonable explanation of recent events in the telecoms industry, so it's hard to decide between the parameter values without delving deeper into the data and the model.

6. *Minus2_a.*

(a) This is easy because we have a constant elasticity demand curve with $\varepsilon = -2$ and a constant marginal cost of \$2. Thus, the Lerner Index form of the monopoly's first order condition tells us that

$$\frac{p - 2}{p} = -\frac{1}{-2} \Rightarrow p^* = 4$$

The demand curve tells us that $x(4) = 1000{\times}4^{-2} = 62.5$. The constant MC is the same as the AC, so there is a profit of $2 per unit, or a total profit of 125.

(b) At $p^* = MC = 2$, the monopoly quantity is

$$x(2) = 1000{\times}2^{-2} = 250$$

The deadweight loss is the area between the price of 2 and 4, but not including the monopoly profit:

$$\int_2^4 1000p^{-2}dp - 125 = -1000{\times}4^{-1} + 1000{\times}2^{-1} - 125 = \$125$$

This is represented by areas A and B in the following figure: