ECON 301, Professor Hogendorn

Problem Set 7 Answers

1. *OilProducers_a.*

   (a) The average costs in the two regions are:

$$\begin{aligned} AC_{ME}(y) &= 20 + 20y^2 \\ AC_{AB}(y) &= 30 + 30y^2 \end{aligned}$$

   It is therefore clear that at $p = 25$, the Alberta firms do not enter the market. The Middle East firms will set $MC(y) = p$:

$$\begin{aligned} c_{ME}(y) &= 20y + 20y^3 \\ MC_{ME}(y) &= 20 + 60y^2 \\ p &= 20 + 60y^2 \\ p - 20 &= 60y^2 \\ y = s(p) &= \frac{p - 20}{60}^{\frac{1}{2}} \end{aligned}$$

   Since there are 20 ME firms, the market supply is

$$S_{ME}(p) = 20\frac{p - 20}{60}^{\frac{1}{2}}$$

   The quantity that would result in a price of 25 is just $S_{ME}(25)$, which is 5.77.

   (b) If we do the same calculations for the AB firms, we find

$$\begin{aligned} c_{AB}(y) &= 30y + 30y^3 \\ MC_{AB}(y) &= 30 + 90y^2 \\ s(p) &= \frac{p - 30}{90}^{\frac{1}{2}} \\ S_{AB}(p) &= 10\frac{p - 30}{90}^{\frac{1}{2}} \end{aligned}$$

From the AC curves, we know the Alberta firms will enter the market when price is 30, so above 30 market supply is

$$S(p) = 20\frac{p-20}{60}^{\frac{1}{2}} + 10\frac{p-30}{90}^{\frac{1}{2}} \text{ if } p \geq 30$$

Below $p = 30$, the maket supply is just $S_{ME}(p)$.

(c) For the price to be 32, we just need to find $S(32)$. This is

$$S(32) = 20\frac{32-20}{60}^{\frac{1}{2}} + 10\frac{32-30}{90}^{\frac{1}{2}} = 8.94 + 1.49 = 10.43$$

2. *OilRefineries_a.*

(a) The Lagrangian is:

$$\begin{aligned}
\max_{L,K,\lambda} \mathcal{L} &= 10L + rK - \lambda(0.147L^{0.3}K^{0.6} - y) \\
\frac{\partial \mathcal{L}}{\partial L} &= 10 - \lambda 0.044L^{-0.7}K^{0.6} = 0 \\
\frac{\partial \mathcal{L}}{\partial K} &= r - \lambda 0.088L^{0.3}K^{-0.4} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 0.147L^{0.3}K^{0.6} - y = 0
\end{aligned}$$

Solving simultaneously we get:

$$\begin{aligned}
\lambda = 224.49L^{0.7}K^{-0.6} & \\
\lambda = r11.56L^{-0.3}K^{0.4} & \qquad L = 0.051rK \\
0.147(0.051rK)^{0.3}K^{0.6} - y = 0 & \;\Rightarrow\; 0.06r^{0.3}K^{0.9} - y = 0 \\
K^{*} = 22.6r^{-0.33}y^{1.11} & \qquad L^{*} = 1.18r^{0.67}y^{1.11}
\end{aligned}$$

(b) First we solve for the short-run labor demand:

$$\mathrm{y} = 0.147\mathrm{L}^{0.3}(359)^{0.6} \Rightarrow \mathrm{L}^{0.3} = 6.8(359)^{-0.6}\mathrm{y}$$

$$\mathrm{L(y)} = 0.0046\mathrm{y}^{3.33}$$

Then short-run total costs are:

$$\mathrm{TC(y)} = 359\mathrm{r} + 10\mathrm{L(y)} = 359\mathrm{r} + 0.046\mathrm{y}^{3.33}$$

And taking the derivative, short-run marginal costs are:

$$\mathrm{MC(y)} = 0.153\mathrm{y}^{2.33}$$

(c) Just set marginal cost equal to price at the output of 15.4:

$$MC(y) = 0.153(15.4)^{2.33} = 89.65 = p$$

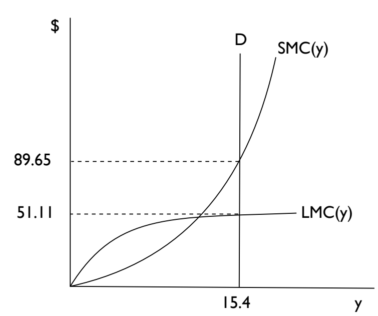
(d) Now find total cost using the conditional factor demands from the Lagrangian:

$$LTC(y) = 1.15 \cdot 22.6(1.15)^{-0.33}y^{1.11} + 10 \cdot 1.18(1.15)^{0.67}y^{1.11}$$
$$LTC(y) = (24.82 + 12.96)y^{1.11}$$

Take the derivative to find LMC, and evaluate at $\mathrm{y} = 15.4$:

$$\mathrm{LMC(y)} = 1.11 \cdot (24.82 + 12.96)\mathrm{y}^{0.11} \qquad \mathrm{MC}(15.4) = 51$$

(e) The key thing about the graph is that the LMC has an exponent of 0.11 while the SMC has an exponent of 2.33. Thus the LMC is increasing concave, while the SMC is increasing convex.



(f) Of course there are many possible answers to this question. Here are some thoughts:

i. Probably the most unrealistic element here is that crude oil is not listed as a factor. That means that crude oil must be rolled in with labor, since labor is the only variable factor in the problem. But the exponent on labor is only 0.3, so it's pretty strange that the share of costs of labor plus crude oil in oil refining would be only 1/3. We could correct this by increasing the exponent on labor and decreasing the one on capital, or to be more realistic add a third factor for crude oil. This change would decrease the importance of capital, which would make the regulatory effects smaller. It would also add some interesting trade-offs between labor, oil, and capital that would be affected by the regulations.

ii. Having only one variable factor in the short run means that there is no cost minimization problem. It suggests that there is only one way to run an oil refinery. In particular, it suggests that refineries never need to be shut down for maintenance, but this is exactly the central issue of the remainder of the problem. If we added some kind of maintenance schedule to the marginal cost, it would not change the answer to part (b) very much, but it would provide more realism and would affect the answers to the remainder of the problem.

iii. There is no direct model of the shut-down costs that prevent the addition of capital. To address this, what is really needed is a dynamic model of shut-down costs over time, complete with a discount rate and expectations of future prices. The decision not to shut down would then be a function of the new parameters rather than just assumed in the problem.

iv. The way the regulations are modeled, the production function remains unchanged, but the cost of capital rises. It is as if every unit of capital requires a pollution-control system. It would be better to explain what the pollution technology does and how it modifies the production function. Presumably capital can still be obtained at (almost) the same rental rate; the issue is that additional capital is acquired for the pollution control systems. This is important because the whole point of the problem is that while the refineries are shut down, the firms might as well add other non-pollution-related forms of capital. Clearly these would *not* be subject to any cost increases.

3. *Coke_a.*

   (a)

$$\frac{p - MC}{p} = \frac{1}{|\varepsilon|} = -\frac{1}{1.2} = 83\%$$

   (b) First find the total cost curve:

$$y = \beta K^2 \Rightarrow K(y) = \beta^{-1/2} y^{1/2} \Rightarrow TC(y) = 20\beta^{-1/2} y^{1/2}$$

   Then substitute the marginal cost into the Lerner index / elasticity formula:

$$\frac{p - 10\beta^{-1/2} y^{-1/2}}{p} = 0.83 \Rightarrow p - 10\beta^{-1/2} y^{-1/2} = 0.83p \Rightarrow$$

$$p = 58.8\beta^{-1/2} y^{-1/2}$$

   From this it is clear that an increase in β *holding y constant* will decrease price. But we really need to count the effect of a change in β on y. To do this, note that any constant elasticity demand curve will have the form $y = AP^{\varepsilon}$, so this one is $y = \beta p^{-1.2}$, and we can sub this in to the formula above to get:

$$p = 58.8\beta^{-1/2} A^{-1/2} P^{0.6} \Rightarrow p^{0.4} = 58.8\beta^{-1/2} A^{-1/2} \Rightarrow$$

$$p^{0.4} = (58.8A)^{-1.25}\beta^{-1.25}$$

Now it is clear that p really does fall with β.