

ECON 301, Professor Hogendorn

Problem Set 9

1. *SubsInc.* Let your utility function be

$$u(x, y) = \sqrt{x} + \sqrt{y}$$

This will give demand functions

$$x(p_x, p_y, m) = \frac{m}{\frac{p_x^2}{p_y} + p_x} \quad y(p_x, p_y, m) = \frac{m}{\frac{p_y^2}{p_x} + p_y}$$

(You can try this for a review of the Lagrangian for the final.)

If $p_x = 1$, $m = 100,000$, and p_y starts at \$1 and then rises, what is the substitution effect and the income effect?

2. *Aisha.* Aisha runs a one-person, ten-cow dairy operation which produces 600 gallons of milk a week. This is her sole source of income. Aisha's utility function is

$$U(x, g) = 60x^2g^4$$

where x = numeraire and g = gallons of milk. Let p_g be the price of milk.

- (a) What is Aisha's demand function for milk?
- (b) Show whether milk is a normal or an inferior good.
- (c) The price of milk is \$4 per gallon. How many gallons of milk does Aisha consume? How much numeraire?
- (d) All the dairies except Aisha's are hit by a tornado, wiping out many cows and causing the price of milk to rise. Break down the corresponding change in Aisha's consumption of milk between the substitution, ordinary income, and full income effects.

3. *Nurses.* Your state is experiencing a nursing shortage and you, as the state nursing czar, are supposed to figure out how to fix the problem. You don't know the specific function, but you know the labor supply of a typical nurse must be some $L^s(w, m)$, and thus the leisure demand function is $R(w, m)$, where w is the wage and m is the "full income." There is no non-labor income.

Nurses cannot supply more than 10 hours of labor per day due to strict regulation in your state, so the endowment of leisure must be $\bar{R} = 10$. Currently the wage is 20, and currently nurses take R^* hours of leisure and C^* worth of consumption.

- (a) Suppose you recommended subsidies that raised the wage to 25. What would be the Marshallian demand for leisure at this new wage? What would be the Slutsky compensated demand for leisure at this new wage?
- (b) Will the nurses definitely work more hours at the new wage? Why or why not?
- (c) Another option would be to give the nurses a lump sum bonus of \$75 per day. What would be the Marshallian and Slutsky compensated demands for leisure under this option?
- (d) Would this work better or worse than the wage increase at alleviating the nursing shortage?

Review problems only, not to turn in:

4. *MrLee.* Mr. Lee is an eccentric millionaire who made his money by manipulating the price of rice in Singapore. He now lives in Middletown, CT, where he purchased a defunct Bradlees department store and converted it to a house. In front of the house is a very large parking lot. Mr. Lee likes to consume large numbers of cars to fill up this parking lot (they can only be the latest model year, so he needs to buy a lot of new cars every year).

Last year the price of Hyundais was \$8,000 and the price of Mercedes was \$45,000. Mr. Lee bought 200 Hyundais and 25 Mercedes. These have now been towed away, and it is time to buy this year's cars. Unfortunately, the price of Hyundais has risen to \$13,000 this year.

The slope of Mr. Lee's Slutsky compensated demand function for Hyundais is -0.001 (i.e. one less Hyundai for each \$1,000 increase in price). The slope of his Engel curve for Hyundais is --0.00001 (i.e. one less Hyundai for each \$100,000 increase in income).

- (a) Using the Slutsky equation, what is the slope of Mr. Lee's Marshallian demand for Hyundais? How many does he buy this year (assuming the linear estimate of slope can be used)?
- (b) Assuming Mr. Lee's income did not change and he spends it all on Hyundais and Mercedes, how many Mercedes does he buy this year?
- (c) Graph Mr. Lee's consumption decisions in the two years using budget lines and indifference curves.
- (d) Which ones of the following describe Hyundais: normal good, inferior good, Giffen good?

5. *Relax*. The demand for relaxation is

$$R(w, p, m) = \frac{1}{4}m + p + \frac{1}{w}$$

w is the wage. p is the price of consumption. There are 16 total hours available, and nonlabor income is 12, so total income is $m = 16w + 12$.

- (a) What is the labor supply curve? Is it backward-bending?
- (b) Denoting (C^*, R^*) as the initial consumption bundle, write the Slutsky equation for relaxation.

- (c) Evaluate the Slutsky equation at $p = 1, w = 0.6$.
- (d) With reference to the income and substitution effects, explain why labor supply curves often bend backward.

Answer to Review Problems:

4. *MrLee_a*. Let h be Hyundais, d be Mercedes, and m be income.

(a) Substituting into the Slutsky equation gives us:

$$\begin{aligned} \frac{\partial h(p_h, p_d, m)}{\partial p_h} &= \frac{\partial h^s}{\partial p_h} - \frac{\partial h(p_h, p_d, m)}{m} h^* \\ \frac{\partial h(p_h, p_d, m)}{\partial p_h} &= -0.001 - (-0.00001)200 \\ &= +0.001 \end{aligned}$$

To estimate the number purchased this year:

$$\frac{\partial h(p_h, p_d, m)}{\partial p_h} \cdot \$5000 = 5$$

so 205 Hyundais this year.

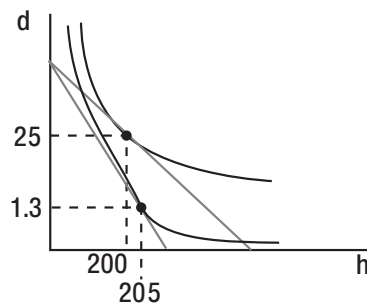
(b) Last year's income must have been

$$m = 8000 \cdot 200 + 45000 \cdot 25 = 2725000$$

This year's budget constraint is:

$$13000 \cdot 205 + 45000 \cdot d^* = 2725000 \Rightarrow d^* = 1.3$$

(c) The graph is:



- (d) Hyundais are an inferior good because their Engel curve slopes down, and they are a Giffen good because the Marshallian demand curve slopes up due to the very strong income effect of a price change.

5. *Relax_a.*

- (a) The labor supply curve is

$$L^S = 16 - R(w, p, 16w + 12) = 16 - 4m - 3 - p - \frac{1}{w}$$

The derivative is $\frac{\partial L^S}{\partial w} = -\frac{\partial R}{\partial w} = -4 + w^{-2}$. This is negative if:

$$\begin{aligned} -4 + w^{-2} &< 0 \\ 4w^2 &> 1 \\ w^2 &> \frac{1}{4} \\ w &> 0.5 \end{aligned}$$

So the supply of labor slopes up for wages less than 0.5, and down for higher wages. Thus labor supply bends backward above 0.5.

- (b) The Slutsky compensated demand is

$$R^S(w) = R(w, p, pC^* + wR^*) = \frac{1}{4}(pC^* + wR^*) + p - \frac{1}{w}$$

From this we can derive the Slutsky equation,

$$\frac{\partial R}{\partial w} = \frac{\partial R^S}{\partial w} + \frac{\partial R}{\partial m}(16 - R^*)$$

$$4 - w^{-2} = 0.25R^* - w^{-2} + 0.25 \cdot (16 - R^*)$$

- (c) Note that $R^* = 5.4 + 1 + 1.67 = 8.067$. Then:

$$4 - 2.78 = 2.017 - 2.78 + 0.25(7.933)$$

$$1.22 = -0.763 + 1.98$$

- (d) The wage is the price of leisure, so as the wage rises, leisure becomes more expensive, and the consumer “buys” less of it due to the substitution effect. However, leisure is also a normal good, meaning it has a positive income elasticity. When the wage rises, the consumer’s endowment of hours is worth more, giving it higher income.

The higher income causes the consumer to buy more of all normal goods, including leisure. At a high enough wage, the endowment income effect becomes large enough to outweigh the ordinary income effect and the substitution effect, so the consumer buys more leisure. Thus, the consumer’s supply of labor falls, and the labor supply curve bends back.