

ECON 301, Professor Hogendorn

Problem Set 9 Answers

1. *SubsInc_a*. For good measure, here's the Lagrangian to start:

$$\begin{aligned}\max_{x,y,\lambda} \mathcal{L} &= \sqrt{x} + \sqrt{y} - \lambda(p_x x + p_y y - m) \\ \frac{\partial \mathcal{L}}{\partial x} &= \frac{1}{2} \frac{1}{\sqrt{x}} - \lambda p_x = 0 \\ \frac{\partial \mathcal{L}}{\partial y} &= \frac{1}{2} \frac{1}{\sqrt{y}} - \lambda p_y = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= p_x x + p_y y - m = 0\end{aligned}$$

Solving simultaneously we get:

$$\begin{aligned}\lambda &= \frac{1}{p_x} \frac{1}{\sqrt{x}} \\ \lambda &= \frac{1}{p_y} \frac{1}{\sqrt{y}} & \lambda = \lambda &\Rightarrow \frac{1}{p_x} \frac{1}{\sqrt{x}} = \frac{1}{p_y} \frac{1}{\sqrt{y}} \\ & & &\Rightarrow x = \frac{p_y^2}{p_x^2} y \\ p_x \frac{p_y^2}{p_x^2} y + p_y y - m &= 0 & \Rightarrow y(p_x, p_y, m) &= \frac{m}{\frac{p_y^2}{p_x} + p_y} \\ & & &x(p_x, p_y, m) = \frac{m}{\frac{p_x^2}{p_y} + p_x} \\ & & &\lambda = \frac{1}{2} \sqrt{\frac{\frac{p_x^2}{p_y} + p_x}{m}} \frac{1}{p_x}\end{aligned}$$

Now, to find the substitution and income effect, we first need to

know the starting point:

$$y^* = y(1, 1, 100000) = \frac{100000}{\frac{1}{1} + 1} = 50000$$

This means the Slutsky compensated demand is

$$y^s(p_y) = y(p_x, p_y, p_x x^* + p_y y^*) = \frac{p_x x^* + p_y y^*}{\frac{p_y^2}{p_x} + p_y}$$

The derivative of the Slutsky compensated demand function is:

$$\frac{\partial y^s}{\partial p_y} = \frac{\partial y}{\partial p_y} + \frac{\partial y}{\partial m} y^*$$

Here that is

$$\frac{\partial y^s}{\partial p_y} = -m \left(\frac{p_y^2}{p_x} + p_y \right)^{-2} \left(\frac{2p_y}{p_x} + 1 \right) + \frac{1}{\frac{p_y^2}{p_x} + p_y} 50000$$

$$\frac{\partial y^s}{\partial p_y} = -100000 \left(\frac{1}{1} + 1 \right)^{-2} \left(\frac{2}{1} + 1 \right) + \frac{1}{\frac{1}{1} + 1} 50000$$

$$\frac{\partial y^s}{\partial p_y} = -100000 \frac{3}{4} + 25000 = -75000 + 25000 = -50000$$

The Slutsky equation rearranges the partial derivative like this:

$$\frac{\partial y}{\partial p_y} = \frac{\partial y^s}{\partial p_y} - \frac{\partial y}{\partial m} y^*$$

So here that is

$$-75000 = -50000 - 25000$$

The the substitution effect is -50000 and the income effect is -25000 .

2. *Aisha_a.*

- (a) Since the utility function is Cobb-Douglas, we know that the demand function will take the form $g(p_g, m) = \frac{2}{3} \frac{m}{p_g}$ where m is the *full income*.

Note that the Cobb-Douglas form means that Aisha always spends 2/3 of her income on milk.

In this case, $m = 600p_g$, so $g(p_g, 600p_g) = \frac{2}{3}600 = 400$.

Since Aisha's income depends only on the price of milk, it turns out that her milk consumption is constant. This unusual result occurs because the endowment income effect will completely cancel the ordinary income and substitution effects. It would not occur if, for example, Aisha had an endowment of x as well.

- (b) Demand for a normal good increases when income increases. Here,

$$\frac{\partial g}{\partial m} = \frac{2}{3p_g} > 0$$

so milk is normal. Note that what we want here is the slope of the Engel curve, which is a *partial* derivative in which m increases but p_g stays constant.

- (c) We already saw that $g(p_g, 600p_g) = 400$ for any p_g . If $p_g = 4$, $m = 600 \times 4 = 2400$. We know that Aisha always spends 2/3 of her income on milk and 1/3 on other goods, so $2400/3=800$ is the amount spent on x . And since $p_x = 1$, $x=800$.

- (d) The derivative of Slutsky compensated demand is

$$\begin{aligned} \frac{\partial g^s}{\partial p_g} &= \frac{\partial g}{\partial p_g} + \frac{\partial g}{\partial m} g^* \\ \frac{\partial g^s}{\partial p_g} &= -\frac{2}{3} \frac{m}{p_g^2} + \frac{2}{3p_g} 400 \\ \frac{\partial g^s}{\partial p_g} &= -\frac{2}{3} \frac{2400}{4^2} + \frac{2}{3 \times 4} 400 \end{aligned}$$

$$\frac{\partial g^s}{\partial p_g} = -100 + 67 = -33$$

Since Aisha has an endowment of $g=600$, the total derivative of her Marshallian demand function is

$$\frac{dg}{dp_g} = \frac{\partial g}{\partial p_g} + \frac{\partial g}{\partial m} 600 = 0$$

(recall we found this was equal to 0 in part a.) Combining this with the Slutsky compensated demand gives

$$\frac{dg}{dp_g} = \frac{\partial g^s}{\partial p_g} - \frac{\partial g}{\partial m} g^* + \frac{\partial g}{\partial m} 600$$

Filling in from above we find

$$0 = -33 - 67 + \frac{2}{3 \times 4} 600 = -33 - 67 + 100$$

Thus the substitution effect is -33 , the ordinary income effect is -67 , and the endowment income effect exactly offsets these at $+100$.

3. Nurses_a.

- (a) The new Marshallian demand is $R(25, 250)$. Note how both the price and the income have risen; this is why we need the Slutsky equation to separate out what is happening. The Slutsky compensated demand at the new wage is

$$R^s(25) = R(25, 25R^* + C^*)$$

- (b) Because this is an endowment problem, we start by taking the derivative of Marshallian demand.

$$\frac{dR}{dw} = \frac{\partial R}{\partial w} + \frac{\partial R}{\partial m} 10$$

Next, we take the derivative of Slutsky compensated demand and rearrange it:

$$\frac{\partial R^s}{\partial w} = \frac{\partial R}{\partial w} + \frac{\partial R}{\partial m} R^*$$

$$\frac{\partial R}{\partial w} = \frac{\partial R^s}{\partial w} - \frac{\partial R}{\partial m} R^*$$

Finally, we can substitute the last equation into the derivative of Marshallian demand to get the three effects:

$$\frac{dR}{dw} = \frac{\partial R^s}{\partial w} - \frac{\partial R}{\partial m} R^* + \frac{\partial R}{\partial m} 10$$

On the right hand side, the first term *must* be negative since it is a compensated demand curve. The derivative $\frac{\partial R}{\partial m}$ is the slope of the Engel curve for leisure – one would assume that it is positive. Thus, the last two terms, the ordinary plus endowment income effect will total up to something positive multiplied by $10 - R^*$, the amount of work the nurses choose to do.

We cannot be sure whether the nurses will work more hours, but we can say that they are less likely to work more if (i) they regard leisure as more of a luxury good and (ii) they already were working most of the possible hours (R^* close to 0).

- (c) The new Marshallian demand would just add the \$75 to income: $R(w, 10w + 75)$. The new Slutsky compensated demand would compensate to $R(w, wR^* + C^* + 75)$.
- (d) Since this is a pure income change, we only need to know the slope of the Engel curve. The effect of the change would be:

$$\frac{\partial R}{\partial m} 75$$

Since leisure is a normal good, this just makes it even more likely that they will take additional leisure. So the bonus is a terrible idea!