ECON 301, Professor Hogendorn

Review Problem Set 10

1. *Aisha.* Aisha runs a one-person, ten-cow dairy operation which produces 600 gallons of milk a week. This is her sole source of income. Aisha's utility function is

$$U(x, g) = 60x^2g^4$$

where x = numeraire and g = gallons of milk. Let p_g be the price of milk.

   (a) What is Aisha's demand function for milk?

   (b) Show whether milk is a normal or an inferior good.

   (c) The price of milk is $4 per gallon. How many gallons of milk does Aisha consume? How much numeraire?

   (d) All the dairies except Aisha's are hit by a tornado, wiping out many cows and causing the price of milk to rise. Break down the corresponding change in Aisha's consumption of milk between the substitution, ordinary income, and endowment income effects.

2. *Nurses.* Your state is experiencing a nursing shortage and you, as the state nursing czar, are supposed to figure out how to fix the problem. You don't know the specific function, but you know the labor supply of a typical nurse must be some $L^s(w, m)$, and thus the leisure demand function is $R(w, m)$, where w is the wage and m is the "full income." There is no non-labor income.

   Nurses cannot supply more than 10 hours of labor per day due to strict regulation in your state, so the endowment of leisure must be $\overline{R} = 10$. Currently the wage is 20, and currently nurses take R^* hours of leisure and C^* worth of consumption.

(a) Suppose you recommended subsidies that raised the wage to 25. What would be the Marshallian demand for leisure at this new wage? What would be the Slutsky compensated demand for leisure at this new wage?

(b) Will the nurses definitely work more hours at the new wage? Why or why not?

(c) Another option would be to give the nurses a lump sum bonus of $75 per day. What would be the Marshallian and Slutsky compensated demands for leisure under this option?

(d) Would this work better or worse than the wage increase at alleviating the nursing shortage?

3. *Relax*. The demand for relaxation is

$$R(w, p, m) = \frac{1}{4}m + p + \frac{1}{w}$$

w is the wage. p is the price of consumption. There are 16 total hours available, and nonlabor income is 12, so total income is $m = 16w + 12$.

(a) What is the labor supply curve? Is it backward-bending?

(b) Denoting (C^*, R^*) as the initial consumption bundle, write the Slutsky equation for relaxation.

(c) Evaluate the Slutsky equation at $p = 1, w = 0.6$.

(d) With reference to the income and substitution effects, explain why labor supply curves often bend backward.

4. *Glendivot*. The Glendivot distillery makes Scotch whisky and then stores it for a period of time before selling it. The market price of a

barrel of Glendivot is

$$V(t) = 480t - 12t^2$$

where t is the number of years of aging.

(a) The distillery produces a private reserve which is consumed only by the owner's family. The objective is to produce the highest possible value Scotch without reference to costs. How long is the private reserve aged?

(b) The distillery also produces its regular product for commercial sale. Its faces an interest rate of 5%, compounded continuously. How many years does it age its regular product?

(c) Suppose the market for Scotch is perfectly competitive and in long run equilibrium with no entry or exit of firms (a dubious assumption, but let's go with it). Assume there is only one cost the distillery incurs: the cost of distilling a barrel's worth in the first place. How much does it cost to distill a barrel? (Note: ignore the private reserve; that is private consumption by the owner.)

5. *Mackenzie.* Mackenzie Rembrandt had a famous ancestor who painted a family portrait that has been handed down to her. The value of this painting increases over time according to the function

$$V(t) = 10t - 2t^2$$

where t is the number of years starting now and V is measured in millions of dollars.

(a) If Mackenzie's objective is to let the painting appreciate to its maximum value and then give it to a museum, when should she give away the painting?

(b) Suppose instead that Mackenzie views the painting as an investment, and she can get a (continuously compounded) interest rate of 7%. When should she sell the painting?

6. *Unionize.* Widget producers have conditional factor demand for labor $L(y) = y^2$. Labor supply is $Ls = \frac{1}{a}w$ where w is the wage and a is a parameter. Set a equal to the first digit in your WesID. Inverse demand for widgets is given by $pd = 5 + b - y$. Set b equal to the third digit in your WesID. Use Mathematica in all of the following, and make sure to comment all your work using section and text cells.

   (a) Suppose there is perfect competition in the widget market and in the labor market. Find the goods market equilibrium, and then find and graph the labor market equilibrium. (5 points)

   (b) Suppose there is a monopoly in the widget market and perfect competition in the labor market. Find the goods market equilibrium, and then find and graph the labor market equilibrium. (5 points)

   (c) Suppose there is perfect competition in the widget market and monopoly in the labor market (i.e. there is a union). Find the goods market equilibrium and then find and graph the labor market equilibrium showing demand, supply, and marginal revenue of the union. (2 bonus points – these points are added to your total points on the exam)

Answers:

5. *Aisha_a.*

   (a) Since the utility function is Cobb-Douglas, we know that the demand function will take the form $g(p_g, m) = \frac{2}{3}\frac{m}{p_g}$ where m is the *full income.*

Note that the Cobb-Douglas form means that Aisha always spends 2/3 of her income on milk.

In this case, $m = 600p_g$, so $g(p_g, 600p_g) = \frac{2}{3}600 = 400$.

Since Aisha's income depends only on the price of milk, it turns out that her milk consumption is constant. This unusual result occurs because the endowment income effect will completely cancel the ordinary income and substitution effects. It would not occur if, for example, Aisha had an endowment of x as well.

(b) Demand for a normal good increases when income increases. Here,

$$\frac{\partial \mathrm{g}}{\partial \mathrm{m}} = \frac{2}{3\mathrm{p_g}} > 0$$

so milk is normal. Note that what we want here is the slope of the Engel curve, which is a *partial* derivative in which m increases but p_g stays constant.

(c) We already saw that $g(p_g, 600p_g) = 400$ for any p_g. If $p_g = 4$, $m = 600Ö4 = 2400$. We know that Aisha always spends 2/3 of her income on milk and 1/3 on other goods, so 2400/3=800 is the amount spent on x. And since $p_x = 1$, x=800.

(d) The derivative of Slutsky compensated demand is

$$\frac{\partial \mathrm{g^s}}{\partial \mathrm{p_g}} = \frac{\partial \mathrm{g}}{\partial \mathrm{p_g}} + \frac{\partial \mathrm{g}}{\partial \mathrm{m}}\mathrm{g}^*$$

$$\frac{\partial \mathrm{g^s}}{\partial \mathrm{p_g}} = -\frac{2}{3}\frac{\mathrm{m}}{\mathrm{p_g^2}} + \frac{2}{3\mathrm{p_g}}400$$

$$\frac{\partial \mathrm{g^s}}{\partial \mathrm{p_g}} = -\frac{2}{3}\frac{2400}{4^2} + \frac{2}{3Ö4}400$$

$$\frac{\partial \mathrm{g^s}}{\partial \mathrm{p_g}} = -100 + 67 = -33$$

Since Aisha has an endowment of g=600, the total derivative of her Marshallian demand function is

$$\frac{dg}{dp_g} = \frac{\partial g}{\partial p_g} + \frac{\partial g}{\partial m}600 = 0$$

(recall we found this was equal to 0 in part a.) Combining this with the Slutsky compensated demand gives

$$\frac{dg}{dp_g} = \frac{\partial g^s}{\partial p_g} - \frac{\partial g}{\partial m}g^* + \frac{\partial g}{\partial m}600$$

Filling in from above we find

$$0 = -33 - 67 + \frac{2}{3Ö4}600 = -33 - 67 + 100$$

Thus the substitution effect is −33, the ordinary income effect is −67, and the endowment income effect exactly offsets these at +100.

6. *Nurses_a.*

   (a) The new Marshallian demand is $R(25, 250)$. Note how both the price and the income have risen; this is why we need the Slutsky equation to separate out what is happening. The Slutsky compensated demand at the new wage is

   $$R^s(25) = R(25, 25R^* + C^*)$$

   (b) Because this is an endowment problem, we start by taking the derivative of Marshallian demand.

   $$\frac{dR}{dw} = \frac{\partial R}{\partial w} + \frac{\partial R}{\partial m}10$$

   Next, we take the derivative of Slutsky compensated demand and rearrange it:

   $$\frac{\partial R^s}{\partial w} = \frac{\partial R}{\partial w} + \frac{\partial R}{\partial m}R^*$$

$$\frac{\partial \mathrm{R}}{\partial \mathrm{w}} = \frac{\partial \mathrm{R^s}}{\partial \mathrm{w}} - \frac{\partial \mathrm{R}}{\partial \mathrm{m}}\mathrm{R}^*$$

Finally, we can substitute the last equation into the derivative of Marshallian demand to get the three effects:

$$\frac{\mathrm{dR}}{\mathrm{dw}} = \frac{\partial \mathrm{R^s}}{\partial \mathrm{w}} - \frac{\partial \mathrm{R}}{\partial \mathrm{m}}\mathrm{R}^* + \frac{\partial \mathrm{R}}{\partial \mathrm{m}}10$$

On the right hand side, the first term *must* be negative since it is a compensated demand curve. The derivative $\frac{\partial R}{\partial m}$ is the slope of the Engel curve for leisure – one would assume that it is positive. Thus, the last two terms, the ordinary plus endowment income effect will total up to something positive multiplied by $10 - R^*$, the amount of work the nurses choose to do.

We cannot be sure whether the nurses will work more hours, but we can say that they are less likely to work more if (i) they regard leisure as more of a luxury good and (ii) they already were working most of the possible hours (R^* close to 0).

(c) The new Marshallian demand would just add the $75 to income: $R(w, 10w+75)$. The new Slutsky compensated demand would compensate to $R(w, wR^* + C^* + 75)$.

(d) Since this is a pure income change, we only need to know the slope of the Engel curve. The effect of the change would be:

$$\frac{\partial \mathrm{R}}{\partial \mathrm{m}}75$$

Since leisure is a normal good, this just makes it even more likely that they will take additional leisure. So the bonus is a terrible idea!

7. *Relax_a.*

(a) The labor supply curve is

$$L^S = 16 - R(w, p, 16w + 12) = 16 - 4m - 3 - p - \frac{1}{w}$$

The derivative is $\frac{\partial L^S}{\partial w} = -\frac{\partial R}{\partial w} = -4 + w^{-2}$. This is negative if:

$$\begin{aligned} -4 + w^{-2} &< 0 \\ 4w^2 &> 1 \\ w^2 &> \frac{1}{4} \\ w &> 0.5 \end{aligned}$$

So the supply of labor slopes up for wages less than 0.5, and down for higher wages. Thus labor supply bends backward above 0.5.

(b) The Slutsky compensated demand is

$$R^S(w) = R(w, p, pC^* + wR^*) = \frac{1}{4}(pC^* + wR^*) + p - \frac{1}{w}$$

From this we can derive the Slutsky equation and substitute in the endowment effect. The result is

$$\frac{dR}{dw} = \frac{dR^s}{dw} + \frac{\partial R}{\partial m}(16 - R^*)$$

$$4 - w^{-2} = 0.25R^* - w^{-2} + 0.25 \cdot (16 - R^*)$$

(c) Note that R^*= 5.4+1+1.67=8.067. Then:

$$4 - 2.78 = 2.017 - 2.78 + 0.25(7.933)$$

$$1.22 = -0.763 + 1.98$$

(d) The wage is the price of leisure, so as the wage rises, leisure becomes more expensive, and the consumer "buys" less of it due to the substitution effect. However, leisure is also a normal good, meaning it has a positive income elasticity. When the wage rises, the consumer's endowment of hours is worth more, giving it higher income.

The higher income causes the consumer to buy more of all normal goods, including leisure. At a high enough wage, the endowment income effect becomes large enough to outweigh the ordinary income effect and the substitution effect, so the consumer buys more leisure. Thus, the consumer's supply of labor falls, and the labor supply curve bends back.

8. *Glendivot_a.*

   (a) To solve $\max_t V(t) = 480t - 12t^2$ find the FOC:

   $$V'(t) = 480 - 24t = 0 \Rightarrow t = \frac{480}{24} = 20$$

   (b) To solve $\max_t PV(V(t)) = e^{-.05t}(480t - 12t^2)$ the FOC is:

   $$\frac{dPV(V(t))}{dt} = (480 - 24t)e^{-.05t} - .05e^{-.05t}(480t - 12t^2) = 0$$
   $$480 - 24t - 24t + 0.6t^2 = 0$$
   $$0.6t^2 - 48t + 480 = 0$$

   Use the quadratic formula or a calculator to find that $t = 11.7$.

   (c) Using the best "technology" available (i.e. $t = 11.7$), there must be a present value of zero when discounted at the proper rate:

   $$PV = -c_B + e^{-.05 \cdot 11.7}(480 \cdot 11.7 - 12(11.7)^2) = 0$$
   $$c_B = \$2,213.56$$

9. *Mackenzie_a.*

   (a) Just maximizing the value of the painting leads to the first order condition:

   $$\frac{dV}{dt} = 10 - 4t = 0 \Rightarrow t = 2.5 \text{ years}$$

(b) In this case, the problem is to maximize $\frac{V(t)}{e^{0.07t}}$, which leads to the first order condition:

$$(10-4t)e^{-0.07t}+(10t-2t^2)(-0.07)e^{-0.07t}=0$$

The exponential terms cancel, and we can simplfy to:

$$0.14t^2-4.7t+10=0$$

Using a calculator (or the quadratic formula), we find

$$t=2.28 \text{ or } 31.29$$

Only the first answer is economically meaningful in this situation.