ECON 301, Professor Hogendorn

Problem Set 3

1. *Sopranos*. There are two goods, numeraire x and cooking c. The price of numeraire is always 1 throughout this problem, and the price of cooking is p_c.

   Mrs. Soprano and Mrs. Bucco both have the same utility function:

   $$u(x,c) = x^{0.8}c^{0.2}$$

   Mrs. Soprano's endowment is $(\omega_{Sx}, \omega_{Sc}) = (100, 10)$. Mrs. Bucco's endowment is $(\omega_{Bx}, \omega_{Bc}) = (10, 10)$.

   With this utility function and these endowments, the demand functions for numeraire for Mrs. Soprano and Mrs. Bucco are

   $$x_S = 0.8\frac{100 + 10p_c}{1} \qquad x_B = 0.8\frac{10 + 10p_c}{1}$$

   (a) If the two women can trade in an Edgeworth Box, what will be the final allocation and what will be the price of cooking?

   (b) Suppose that the "powers than be" decide that this final allocation is not all right. They want the final allocation to be $(x_B, c_B) = (66, 12)$. Note that (66,12) IS on the contract curve. What lump sum taxes and subsidies on the numeraire are necessary to make this happen? Illustrate with an Edgeworth Box diagram.

2. *Pate*. There are two goods, beef (B) and goose liver pate (G). The typical French person has an endowment of $\omega_B = 50, \omega_G = 50$ and a utility function $U(B,G) = B^{0.3}G^{0.7}$. The typical American has an endowment of $\omega_B = 70, \omega_G = 30$ and a utility function $U(B,G) =$

$B^{0.8}$. Note that the typical American simply does not receive utility from the pate.

(a) What is the typical French and American MRS in (B,G) space at the endowment points?

(b) Draw an Edgeworth box and show indifference curves for each type of consumer. Show the core and the contract curve.

3. *CokePepsi.* The income elasticity demand for Coke is $\epsilon_m^c = 0.58$. For Pepsi, the income elasticity is $\epsilon_m^p = 1.38$ at the current equilibrium points

(a) Which apply to Coke and Pepsi: normal, inferior, luxury, necessity? Why?

(b) Suppose in equilibrium, a person buys 1 bottle of each drink. Draw the Engel Curves for Coke and Pepsi. What is the slope of each Engel Curve at the equilibrium point?

(c) Suppose we calculated a cross-price elasticity of Coke for Pepsi:

$$\epsilon_{cp} = \frac{\partial q_{coke}}{\partial p_{pepsi}} \frac{p_{pepsi}}{q_{coke}}$$

What sign do you expect? Why?

(d) Suppose the demand function for Coke is $q_{coke}(p_{coke}, p_{pepsi}, m)$. Write the total differential of this function.

Review Problems, not to turn in:

4. *Urp.* The residents of Urp consume only pork chops (X) and Coca-Cola (Y). The utility function of the typical resident is given by

$$U(X,Y) = \sqrt{XY}$$

In 2006, the price of pork chops in Urp was \$1 each; Cokes were also \$1 each. The typical resident consumed 40 pork chops and

40 Cokes (saving is impossible in Urp). In 2007, swine fever hit Urp, and pork chop prices rose to \$4; the Coke price remained unchanged. At these new prices, the typical Urp resident consumed 20 pork chops and 80 Cokes.

(a) What was the change in utility from 2006 to 2007? (Just plug into the utility function, don't use differentials.)

(b) What was the Laspeyres price index for 2007?

(c) What was the Paasche price index for 2007?

(d) What do you conclude about the ability of price indices to measure changes in welfare? (Hint: calculate how much income the typical Urp resident had in 2006 and 2007.)

5. *AishaMrLee.* Aisha's utility function is

$$u(G,V) = G^{0.7}V^{0.3}$$

and Mr. Lee's utility function is

$$u(G,V) = G^{0.9}V^{0.1}$$

Aisha has 20 ounces of G and 10 ounces of V. Mr. Lee has 15 ounces of G and 15 ounces of V. This is all the G and V there is in the world, and there are no other people to trade with.

(a) Calculate the MRS in (G,V) space for both consumers at the endowment point.

(b) Draw an Edgeworth box showing the endowment and indifference curves of the consumers. (The indifference curves do not have to be plotted to match the utility function perfectly.)

(c) Assume that Aisha and Mr. Lee can trade at a market price as price-takers. If we set G as the numeraire, what is the price of V? What is the final allocation of G and V?

(d) Show the trading in your diagram.

6. *Pareto.* Is it possible to have a Pareto efficient allocation where someone is worse off than he is at an allocation that is not Pareto efficient? Illustrate with an Edgeworth Box.

7. *RichAndPoor.* A very rich person and a very poor person are going to trade in an Edgeworth box. The rich person is named Ms. 1 and her origin is the lower left corner. The poor person is named Mr. 2 and his origin is the upper right hand corner. The two people will trade good y (on the vertical axis) and good x (on the horizontal axis). Ms. 1 has the entire endowment of good x, and there is a lot of that good. Mr. 2 has the entire endowment of good y, but there is not that much of it. Both people's indifference curves indicate that good y doesn't bring very much utility compared to good x.

   (a) Draw the Edgeworth box, showing the endowment point, indifference curves, and the contract curve. What is the Walrasian equilibrium? Is it efficient?

   (b) Suppose the government values equality and wants the final outcome of trading to be the allocation approximately in the center of the box. Show a government price control that forces the center point to be in the budget sets of both consumers. How does this change the Walrasian equilibrium? Is "equality" achieved? Is this solution efficient.

   (c) Can the government use the Second Fundamental Theorem of Welfare Economics to improve on part (b)?

Answers to Review Problems:

4. *Urp_a.*

   (a) $U_{2006} = \sqrt{40 \cdot 40} = 40$, $U_{2007} = \sqrt{20 \cdot 80} = 40$

   (b) $\frac{4 \cdot 40 + 1 \cdot 40}{1 \cdot 40 + 1 \cdot 40} = 2.5$

   (c) $\frac{4 \cdot 20 + 1 \cdot 80}{1 \cdot 20 + 1 \cdot 80} = 1.6$

   (d) We know that in actual fact, utility was unchanged and the new income in 2007 must have been $4 \cdot 20 + 1 \cdot 80 = 160$ which was twice the income of $1 \cdot 40 + 1 \cdot 40 = 80$ in 2000. Thus, Laspeyres overstated the amount of income needed to keep utility constant, and Paasche understated it.

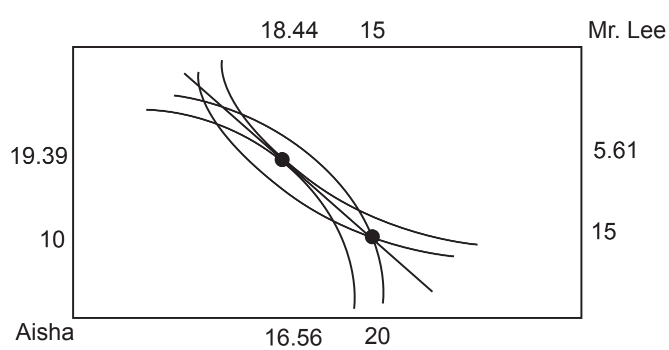
5. *AishaMrLee_a.*

   (a)

$$MRS_A = -\frac{\frac{\partial u}{\partial G}}{\frac{\partial u}{\partial V}} = -\frac{.7G^{-.3}V^{.3}}{.3G^{.7}V^{-.7}} = -\frac{7}{3}\frac{V}{G} = -\frac{7}{3}\frac{10}{20} = -\frac{7}{6}$$

$$MRS_L = -\frac{\frac{\partial u}{\partial G}}{\frac{\partial u}{\partial V}} = -\frac{.9G^{-.1}V^{.1}}{.1G^{.9}V^{-.9}} = -9\frac{V}{G} = -9\frac{15}{15} = -9$$

   (b) and (d)



(c) We have seen before that the demand functions for a Cobb-Douglas will produce the following results:

$$\begin{aligned} G_A &= 0.7\frac{m}{p_G} = 0.7\frac{20+10p_V}{1} \\ G_L &= 0.9\frac{m}{p_G} = 0.9\frac{15+15p_V}{1} \end{aligned}$$

Thus, the market equilibrium condition is:

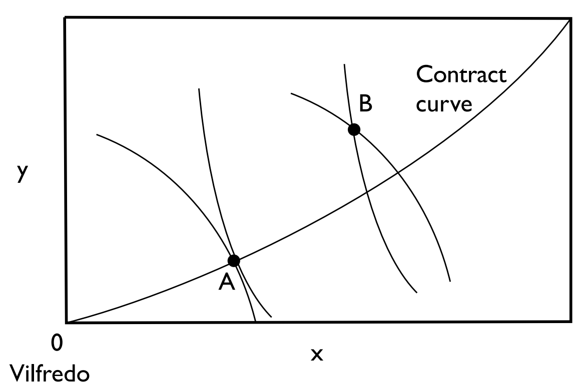
$$\begin{aligned} G_A + G_L &= 35 \\ 27.5 + 20.5p_V &= 35 \end{aligned}$$

Solving this gives

$$20.5p_V = 7.5 \Rightarrow p_V = 0.366$$

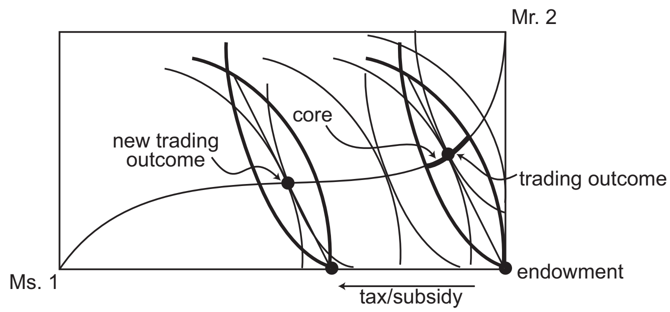
Thus $G_A = 16.56$, $G_L = 18.44$, $V_A = 19.39$, $V_L = 5.61$.

6. *Pareto_a.* Yes, Pareto efficiency says that it is not possible to make one person better off without making another person worse off. But that does not preclude making one person better off *and* making the the other worse off. For example, in the graph Vilfredo is better off at point B than point A, even though B is not on the contract curve and not Pareto-efficient while A is.



7. *RichAndPoor_a.*

(a) To show the relatively low value both people put on good y, we need a large MRS (steeply sloped indifference curves).



(b) Although the center point would then be feasible from the point of view of the budget line, it would not be a Walrasian equilibrium. The indifference curves of the two consumers would be tangent at two different points along this budget line, so there would not be a market-clearing equilibrium. Some gains from trade would be lost, and the center point would not actually be achieved. In any case, unless the center point lies exactly on the contract curve, it is not efficient, since the consumers can Pareto-improve on it.

(c) The second welfare theorem says that any point on the contract curve can be supported as a Walrasian equilibrium provided the consumers begin at the proper endowment point. In the graph, the government could lump-sum-redistribute good x from Mr. 2 to Ms. 1. Then the agents can trade at free-market prices and come as close to the center of the box as the contract curve will allow.