

ECON 301, Professor Hogendorn

Problem Set 5

1. *Skype*. Suppose there are 20 (think 20 million) people who might want to use Skype. They will download Skype to their computers according to the function

$$n = 20 - \hat{v}$$

where  $\hat{v}$  is the quality adjusted price for Skype. Skype creates a network effect  $\theta n$  for a type- $\theta$  user, so if  $p$  is the price of Skype software, all users with  $\theta n \geq p$  will buy Skype.

- (a) Write down Skype's inverse demand curve ( $p$  as a function of  $n$ ) and draw it on a graph.
  - (b) Suppose that Skype's marginal cost function is  $MC = 10 - n$  for  $n < 10$  and  $MC = 0$  for all greater levels of  $n$ . (This is because for a large enough user base, the users buy so many complementary products that there is no net cost to provide them with regular Skype service.) Draw this on your graph.
  - (c) Discuss what equilibria may emerge in this market. Which equilibria are stable and which unstable?
2. *Sigma*. Consider the production function

$$f(L, K) = (L^{0.25} + K^{0.25})^3$$

- (a) What is the formula for MRTS?
- (b) Does this production function exhibit decreasing, constant, or increasing returns to scale?
- (c) What is the elasticity of substitution?

3. Do problem 11.6 parts a and b from the Nicholson reading.

**Review Problems, not to turn in:**

4. *LibraryTax*. There are 10,000 residents of a town, each of whom has an income of \$50,000. There is a numeraire good  $x$  and a library which is a public good; the number of books in the library is  $B$ . Each resident has a Cobb-Douglas utility function

$$u(x, B) = x^{0.99} B^{0.01}$$

The library can buy books for 20 each.

- (a) What is the private MRS of one resident for numeraire and library books in  $(B, x)$  space (yes, this is a little surprising that numeraire is on the vertical axis, but since we're mainly concerned with books here, we need to make them the primary focus). How many books would a single resident put in the library?
- (b) Use the Samuelson condition to find the optimal number of books for the library.
5. Do problem 11.1 from the Nicholson reading.

**Answer to Review Problems:**

4. *LibraryTax\_a*.

- (a) The marginal utilities are

$$\frac{\partial u}{\partial x} = 0.99x^{-0.01}(B)^{0.01} \quad \text{and} \quad \frac{\partial u}{\partial B} = 0.01x^{0.99}(B)^{-0.99}$$

Taking the ratio of these partials gives the private MRS. Setting it equal to the slope of the budget line gives

$$MRS = -\frac{1}{99} \frac{x}{B} = -p_b = -20$$

This gives  $x = B$ . To see how much each resident want to buy, we need to plug this into the budget constraint:

$$p_x x + p_B B = m \Rightarrow 1,980B + 20B = 50,000$$

So each resident will vote for a library containing just 25 books.

- (b) To find the social optimum, use the Samuelson condition and add up all the private MRS's. Since each person is the same, we just multiply the private MRS by 10,000 and set equal to the price ratio:

$$\sum MRS = -10,000 \cdot \frac{1}{99} \frac{x}{B} = -20$$

This gives  $x = 0.198B$ . Now substituting into the budget constraint gives

$$p_x x + p_B B = m \Rightarrow 0.198B + 20B = 50,000$$

So now each person will vote for a library of 2,475 books.

#### 5. *Nicholson11.1\_a*

- (a) The graph is upward sloping and concave.
- (b)  $AP_L = \frac{100\sqrt{L}}{L} = 100L^{-0.5}$ . This is clearly decreasing in  $L$ .
- (c)  $\frac{dq}{dL} = 100 \cdot 0.5L^{-0.5} = \frac{50}{\sqrt{L}}$ . Because the total product of  $L$  is concave everywhere, it is always the case that  $MP_L$  is diminishing. That means that each additional worker always pulls down the average product.