ECON 301, Professor Hogendorn

Problem Set 6

1. *OilRefineries*. A common argument against environmental regu-
   lations is that they will act like a tax and raise the price of goods.
   When there is a lumpiness to costs, however, this may turn out not
   to be true. For example, in 1990 several amendments were passed
   to the Clean Air Act which required oil refineries to significantly
   upgrade their capital. The surprising result was that the refining
   industry increased its total supply and the market prices of refined
   products actually fell, *ceteris paribus*. Here's how this can happen.

   Suppose the demand for oil in the U.S. is 15.4 million barrels per
   day, and is perfectly inelastic. Suppose we can treat all U.S. oil re-
   fineries as a single firm with production function

   $$f(L,K) = 0.147L^{0.3}K^{0.6}$$

   Let $w = 10$, but keep r as a parameter.

   (a) Use the Lagrangian to find the conditional factor demands
       for labor and capital.

   (b) Suppose that in 1989, the refining industry had $K = 359$ and
       was stuck in the short-run. Show that the short-run marginal
       cost curve for the whole industry is

       $$MC(y|K = 359) = 0.153y^{2.33}$$

   (c) During the early 1990s, oil refineries did not move into the
       long run because there were additional costs associated with
       shutting down the refineries in order to replace the capital.
       If the firms remain stuck in the short run, what would be the

price of oil assuming perfectly competitive behavior? What is the firm's average total cost if $r = 1$.?

(d) Now suppose that the government regulation raises the cost of capital to $r = 1.15$ and it forces the firms to move to the long run so they can adjust their capital. If demand is still 15.4, what would be the price of oil? Now what would be the firm's average total cost?

(e) Graph the supply-demand equilibrium of parts (c) and (d).

(f) For each part above, a-d, what, in your opinion, is the most limiting assumption underlying the choice of modeling approach. For example for part (a), your answer might be "It is not realistic to treat all refineries as a single firm because." (This example is not a very good answer, however.) For each limitation, say what is being being oversimplified, and how addressing the concern might be expected to change the answer.

2. *LRSR*. A firm has the following production function:

$$f(K, L) = \sqrt{K} + \sqrt{L}$$

where K is capital and L is labor. The price of K is 1 and the price of L is 2.

(a) Write the Lagrangian and find the first order conditions.

(b) Use Mathematica to solve simultaneously and confirm that the long-run conditional factor demand functions for K and L are

$$K(y) = \frac{4y^2}{9}$$

$$L(y) = \frac{y^2}{9}$$

(c) Use Mathematica to find the LRTC, LRMC, and LRAC.

3. *Luxray.* Luxray Inc. is a firm with cost function $TC(y) = y^2 + 10$. This firm is a perfectly competitive price-taker in a market where $p = 100$.

   (a) Write down Luxray's average cost function, average variable cost function, and marginal cost function. Why does Luxray produce $y^* = 50$ in short-run equilibrium?

   (b) Find Luxray's net profit at $y^* = 50$. Write it down three ways and verify that they are all equal: (i) total revenue minus total cost, (ii) net profit margin times quantity, (iii) operating profit margin times quantity, minus fixed costs.

   (c) Suppose that the cost functions we have been working with *are* the long-run cost functions. However, firms may enter or exit this market freely in the long run. Should Luxray management expect to produce more or less than 50 units as the industry moves toward long-run equilibrium? Explain using a graph.

   Review problems only, not to turn in:

4. *Tequila.* The spirit tequila is produced by distilling the fermented juice of the agave plant. True tequila can only be made from agave grown in the officially denominated tequila region in the environs of Tequila, Mexico. An agave plant takes 8 years to reach maturity, and the region was hit by a freak frost in 1997 that killed many agave plants.

   Suppose that tequila is produced according to the following production function:
   $$f(x, a) = 143x^{0.05}a$$
   a is metric tons of agave and x is a composite factor including labor, oak casks, grinding equipment, and so forth. The idea behind

this production function is that if $x = 1$, each ton of agave produces 143 liters of tequila, but x could be adjusted to change this amount.

Let the price of X be 400 pesos and the price of a metric ton of agave is 1,000 pesos.

   (a) What is the long-run cost curve for tequila?

   (b) If distillers set LRAC(q)=6, how much a do they use?

   (c) Suppose that after the freeze, 90% of the amount of a from part (b) remains available, and so it becomes a fixed factor. If, nevertheless, distillers want to produce the same ouput, what is the cost?

5. *VisaDiscover*. Visa and Discover are considering the introduction of debit cards. Both firms have the same production function $f(L, K) = L^{.8}K^{.3}$. Labor and capital both cost $10 per unit.

   (a) What is the long run total cost curve for either company? Use the Lagrangian to show your answer.

   (b) Assume K is fixed in the short run. Confirm that the short-run total cost curve is $TC(y|K) = 10K + 10K^{-0.375}y^{1.25}$.

6. *Technologies*. Suppose there are three technologies for providing a new Internet service, and one of them will eventually emerge as clearly superior to the other two. All three technologies use the factors S for servers and B for bandwidth, and the factor prices are $w_S = w_B = 1$.

Technology A has production function $F(S, B) = S^{0.7}B^{0.7}$ but S must be set to 10 units and can never be changed.

Technology B has production function $F(S, B) = S^{0.6}B^{0.6}$ and both factors can be freely varied.

Technology C has production function $F(S, B) = S^{0.3}B^{0.6}$ and both factors can be freely varied.

Which technologies have economies of scale? Which have diseconomies of scale?

7. *Consulting.* Technology can improve labor productivity. One might be concerned that this could be bad for workers since fewer would be needed to produce the same output. Displaced workers might have to move to another industry. To think about this, suppose an industry has the production function $f(L) = \alpha L^{0.5}$. The conditional factor demand is thus $L(y) = \left(\frac{1}{\alpha}\right)^2 y^2$. Let $w_L = 1$ throughout this whole problem (i.e. overall labor market equilibrium is unaffected by the changes in the industry we examine here). Suppose there is a fixed cost to start a firm which is $F = 2500$. The cost function is thus

$$c(y) = \left(\frac{1}{\alpha}\right)^2 y^2 + 2500$$

Note that this is both the short-run and the long-run cost function; the only difference is that in the long run a firm can exit or enter the industry.

Suppose that demand in the industry is given by $X(p) = 60000p^\epsilon$. Elasticity ϵ can take on two values: -0.5 and -1.5. Answer the following for each of these values:

(a) Suppose that initially $\alpha = 100$ and the industry is in long-run perfectly competitive equilibrium. How many firms are there? What is the total number of workers?

(b) Suppose that improved technology causes a change to $\alpha = 160$. In the short run (i.e. with the number of firms fixed) what is the new total number of workers?

(c) In the long run, the number of firms will adjust to the new situation. What is the new number of firms and the new total number of workers?

(d) Describe in words what an individual worker would experience during parts (a)-(c). For example, your description might read "First I noticed that my firm hired a few new people. Later, some of our competitors went out of business. Most of the people who worked for those firms came to work at my firm and our remaining competitors, but a few had to get jobs in another industry." Remember to do this for both values of elasticity, and discuss which elasticity is preferable for the workers.

Answers to Review Problems:

4. *Tequila_a.*

(a) Let's use the MRTS to find the answer:

$$\text{MRTS} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial a}} = -\frac{0.05 \cdot 143 a x^{-0.095}}{143 x^{0.05}} = -0.05 \frac{a}{x}$$

$$-0.05 \frac{a}{x} = -\frac{400}{1000} \Rightarrow \frac{a}{x} = 8 \Rightarrow a = 8x$$

Then $q = 143(8x) \cdot x^{0.05} = 1144 x^{1.05}$ and inverting this we find that to produce q, the conditional factor demand is

$$x(q) = 0.0012 q^{0.95}$$

Thus the cost of q is:

$$TC(q) = 400 \cdot 0.0012 q^{0.95} + 1000 \cdot 8 \cdot 0.0012 q^{0.95} = 10.27 q^{0.95}$$

(b) The long run average cost is

$$AC(q) = \frac{TC(q)}{q} = 10.27 q^{-0.05}$$

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If $AC(q) = 6$, then

$$10.27q^{-0.05} = 6 \Rightarrow q = 46433$$

Now we use the conditional factor demand:

$$x(46433) = 32.56 \qquad a(46433) = 260.5$$

(c) The short-run problem is:

$$f(0.9 \cdot 260.5, x) = 143 \cdot 234.45 \cdot x^{0.05}$$
$$\Rightarrow q = 33526.35 x^{0.05}$$

To produce output 46433, the producers need:

$$46433 = 33526.35 x^{0.05} \Rightarrow x = 674.23$$

Thus, the new total cost is

$$TC(46433|a = 234.45) = 1000 \cdot 234.45 + 400 \cdot 674.23 = 504142.82$$

Thus, the freeze caused a huge cost increase, assuming production did not change.

5. *VisaDiscover_a.*

(a)

$$
\begin{aligned}
\max_{K,L,\lambda} \mathcal{L} &= 10K + 10L - \lambda(K^{0.3}L^{0.8} - y) \\
\frac{\partial \mathcal{L}}{\partial K} &= 10 - \lambda 0.3 K^{-0.7} L^{0.8} = 0 \\
\frac{\partial \mathcal{L}}{\partial L} &= 10 - \lambda 0.8 K^{0.3} L^{-0.2} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= K^{0.3} L^{0.8} - y = 0
\end{aligned}
$$

Solving simultaneously we get:

$$\lambda = 33.33K^{0.7}L^{-0.8}$$

$$\lambda = 12.5K^{-0.3}L^{0.2} \qquad K = 0.375L$$

$$(0.375L)^{0.3}L^{0.8} - y = 0 \quad \Rightarrow \quad 0.75L^{1.1} - y = 0$$

$$L^* = 1.3y^{0.91} \qquad K^* = 0.5y^{0.91}$$

$$TC(y) = 10K^* + 10L^* = 13y^{0.91} + 5y^{0.91} = 18y^{0.91}$$

(b)

$$y = K^{0.3}L^{0.8} \Rightarrow L^{0.8} = K^{-0.3}y \Rightarrow L(y|K) = K^{-0.375}y^{1.25}$$

$$TC(y|K) = 10K + 10L(y|K) = 10K + 10K^{-0.375}y^{1.25}$$

6. *Technologies_a.* Technology B always has economies of scale at all output levels because the exponents add to more than 1.

   Technology C has exponents that add to less than 1, so there are diseconomies of scale at all levels of output.

   For Technology A, we can rewrite the production function as $F(B) = 5B^{0.7}$. Thus, $B = 0.1y^{10/7}$ and $c(y) = 10 + 0.1y^{10/7}$. The average cost is $AC(y) = \frac{10}{y} + 0.1y^{3/7}$. The marginal cost is $MC(y) = \frac{1}{7}y^{3/7}$. To find the bottom of the U of average cost

$$
\begin{aligned}
AC(y) &= MC(y) \\
\frac{10}{y} + 0.1y^{3/7} &= \frac{1}{7}y^{3/7} \\
\frac{10}{y} &= 0.04y^{3/7} \\
10 &= 0.043y^{10/7} \\
y &= 45.34
\end{aligned}
$$

So for $y < 45.34$ there are economies of scale, but for greater y there are diseconomies.

7. *Consulting_a.*

   (a) The average and marginal cost curves in this case are:

   $$AC(y) = \frac{2500}{y} + 0.0001\,y \qquad\qquad MC(y) = 0.0002\,y$$

   Thus, the minimum average cost is $\frac{2500}{y} + 0.0001\,y = 0.0002\,y \Rightarrow$ $y_{LR} = 5000$. At this output, the amount of labor employed by each firm is $L(5000) = 2500$.

   The marginal cost of this output level is $MC(5000) = 1$, and since perfectly competitive firms set price equal to marginal cost, we have $p_{LR} = 1$. This is the long run supply curve.

   Equating supply to demand, we find the demand at $p = 1$, which is $60000 \cdot 1^e = 60000$.

   The number of firms in the market must therefore be $N = \frac{60000}{5000} = 12$. Since each firm employs 2500 workers, total employment is 30000.

   (b) Now that $\alpha = 160$, the conditional factor demand is $L(y) = 0.00004\,y^2$ and the total cost function is $c(y) = 0.00004\,y^2 + 2500$. Thus, the new marginal cost curve and the short-run firm supply curve is:

   $$MC(y) = 0.00008\,y \qquad\qquad s(p) = 12500p$$

   Since the number of firms cannot change in the short run, there are still 12 of them, so the market supply curve is just $12s(p)$, and setting supply equal to demand gives us:

   $$
   \begin{aligned}
   60000p^\epsilon &= 12 \cdot 12500p \\
   p^{\epsilon-1} &= 2.5 \\
   p &= 2.5^{\frac{1}{\epsilon-1}}
   \end{aligned}
   $$

$$p = (0.54, 0.69) \text{ when } \epsilon = (-0.5, -1.5)$$

$$X(p) = (81650, 104683) \text{ when } \epsilon = (-0.5, -1.5)$$

$$y = (6804, 8724) \text{ when } \epsilon = (-0.5, -1.5)$$

$$L(y) = (1852, 3044) \text{ when } \epsilon = (-0.5, -1.5)$$

$$NL(y) = (22224, 36528) \text{ when } \epsilon = (-0.5, -1.5)$$

(c) With $\alpha = 160$, the average and marginal cost curves are:

$$AC(y) = \frac{2500}{y} + 0.00004\,y \qquad MC(y) = 0.00008\,y$$

Thus, the minimum average cost is $\frac{2500}{y} + 0.00004\,y = 0.00008\,y \Rightarrow$ $y_{LR} = 7906$. At this output, the amount of labor employed by each firm is $L(7906) = 2500$. (Note this is the same as before, which occurs because we have only changed the coefficient on the production function.)

The marginal cost of this output level is $MC(7906) = 0.632$, and since perfectly competitive firms set price equal to marginal cost, we have $p_{LR} = 0.632$. This is the long run supply curve. Equating supply to demand, we find:

$$p = (0.632, 0.632) \text{ when } \epsilon = (-0.5, -1.5)$$

$$X(p) = (75473, 119420) \text{ when } \epsilon = (-0.5, -1.5)$$

$$N = (9.54, 15.1) \text{ when } \epsilon = (-0.5, -1.5)$$

$$L(y) = (2500, 2500) \text{ when } \epsilon = (-0.5, -1.5)$$

$$NL(y) = (23866, 37750) \text{ when } \epsilon = (-0.5, -1.5)$$

(d) Case of $\epsilon = -0.5$: I never should have taken the job at Sprint. Everything was fine until stupid researchers at Bell Labs and Nortel introduced the new technology. There was overcapacity everywhere, and Sprint laid off about 25% of its workforce. After a while, Global Crossing, Williams, and Worldcom filed

for bankruptcy. But now that there's been some consolidation, Sprint is doing a little better, and it looks like the laid-off workers will be rehired. My friends at Global Crossing are out of luck though – there won't be any telecoms jobs for them.

Case of $\epsilon = -1.5$: When I started at Nortel, it seemed like a sleepy firm, but then this great new technology came along. Nortel grew really fast, and we hired all kinds of new people. Fortunately, I saw that the good times couldn't last, so I cashed in my stock options and moved to a startup. It's a good thing, because Nortel laid off most of the people it hired. My new firm's hanging in there, but it's not like the boom times.

Clearly, the $\epsilon = -1.5$ is preferable, but note that even then there were some layoffs in this model. Also note that both examples provide a somewhat reasonable explanation of recent events in the telecoms industry, so it's hard to decide between the parameter values without delving deeper into the data and the model.