ECON 301, Professor Hogendorn

# Problem Set 9

1. *KmartWalMart.* Suppose that Kmart and Wal-Mart both produce a composite output $q$ which is some measure of floorspace and sales. Kmart's and Wal-Mart's cost curves are

$$c(q_K) = q_K \qquad c(q_W) = 0.7q_W$$

The market demand for the composite good is $p(Q) = 500 - 4(q_K + q_W)$. The firms are Cournot competitors. What is the price and what are Kmart's and Wal-Mart's market shares and profits?

2. *SubsInc.* Let your utility function be

$$\mathrm{u(x, y)} = \sqrt{\mathrm{x}} + \sqrt{\mathrm{y}}$$

This will give demand functions

$$\mathrm{x(p_x, p_y, m)} = \frac{\mathrm{m}}{\frac{\mathrm{p_x^2}}{\mathrm{p_y}} + \mathrm{p_x}} \qquad \mathrm{y(p_x, p_y, m)} = \frac{\mathrm{m}}{\frac{\mathrm{p_y^2}}{\mathrm{p_x}} + \mathrm{p_y}}$$

(You can try this for a review of the Lagrangian for the final.)

If $p_x = 1$, $m = 100,000$, and $p_y$ starts at \$1 and then rises, what is the substitution effect and the income effect?

3. *BigMacs.* You buy a lot of Big Macs. You are also on your town's zoning board, and McDonald's REALLY wants to build a new restaurant there. McDonald's raises the price of Big Macs from \$3 to \$4. Your demand for Big Macs is $x(p, m) = 0.01\frac{m}{p^{1.5}}$. Your income $m$ is \$50,000.

You complain about the price increase, and subtly hint that it could affect your zoning decision. In response, McDonald's sends a representative who will compensate you with coupons for free Big

Macs (fractional coupons are allowed). Here are three possible ways to compensate you:

(a) Calculate a Laspeyres price index, calculate the additional income you would need according to the price index, and divide that amount by \$4 to get the number of Big Mac coupons.

(b) Use Slutsky income-compensated demand to calculate the substitution effect, and give that many Big Mac coupons.

(c) Use Marshallian demand to calculate the change in demand, and give that many Big Mac coupons.

## Review problems only, not to turn in:

4. *Varian27.1.* Varian, Chapter 27, Review Question #1.

5. *MrLee.* Mr. Lee is an eccentric millionaire who made his money by manipulating the price of rice in Singapore. He now lives in Middletown, CT, where he purchased a defunct Bradlees department store and converted it to a house. In front of the house is a very large parking lot. Mr. Lee likes to consume large numbers of cars to fill up this parking lot (they can only be the latest model year, so he needs to buy a lot of new cars every year).

   Last year the price of Hyundais was \$8,000 and the price of Mercedes was \$45,000. Mr. Lee bought 200 Hyundais and 25 Mercedes. These have now been towed away, and it is time to buy this year's cars. Unfortunately, the price of Hyundais has risen to \$13,000 this year.

   The slope of Mr. Lee's Slutsky compensated demand function for Hyundais is -0.001 (i.e. one less Hyundai for each \$1,000 increase in price). The slope of his Engel curve for Hyundais is –0.00001 (i.e. one less Hyundai for each \$100,000 increase in income).

(a) Using the Slutsky equation, what is the slope of Mr. Lee's Marshallian demand for Hyundais? How many does he buy this year (assuming the linear estimate of slope can be used)?

(b) Assuming Mr. Lee's income did not change and he spends it all on Hyundais and Mercedes, how many Mercedes does he buy this year?

(c) Graph Mr. Lee's consumption decisions in the two years using budget lines and indifference curves.

(d) Which ones of the following describe Hyundais: normal good, inferior good, Giffen good?

## Answer to Review Problems:

4. *Varian27.1_a*. First we need to set up the profit function for firm 1 and take the first order condition to get firm 1's best response function:

$$\max_{y_1} \pi_1 = (a - b(y_1 + y_2))y_1 - cy_1$$

Solving the first order condition gives:

$$\frac{\partial \pi_1}{\partial y_1} = (a - b(y_1 + y_2)) - by_1 - c = 0 \Rightarrow y_1 = \frac{a - by_2 - c}{2b}$$

The problem is identical for firm 2, so we also know that firm 2 will have a best response function

$$y_2 = \frac{a - by_1 - c}{2b}$$

A Cournot-Nash equilibrium is the quantity-pair such that both firms are playing their best responses simultaneously, so neither will want to deviate unilaterally. To find it, we just solve the best response functions simultaneously:

$$\begin{aligned} y_1 &= \frac{a-c}{2b} - \frac{a-by_1-c}{4b} \\ y_1\left(1-\frac{1}{4}\right) &= \frac{a-c}{4b} \\ y_1 &= \frac{a-c}{3b} \end{aligned}$$

Since the problem is symmetric, $y_2$ will be the same.

5. *MrLee_a*. Let $h$ be Hyundais, $d$ be Mercedes, and $m$ be income.

   (a) Substituting into the Slutsky equation gives us:

   $$\begin{aligned} \frac{\partial h(p_h, p_d, m)}{\partial p_h} &= \frac{\partial h^s}{\partial p_h} - \frac{\partial h(p_h, p_d, m)}{m} h^* \\ \frac{\partial h(p_h, p_d, m)}{\partial p_h} &= -0.001 - (-0.00001)200 \\ &= +0.001 \end{aligned}$$

   To estimate the number purchased this year:

   $$\frac{\partial h(p_h, p_d, m)}{\partial p_h} \cdot \$5000 = 5$$

   so 205 Hyundais this year.

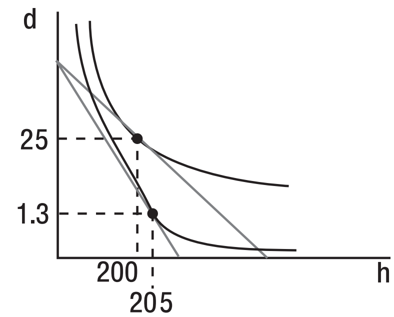
   (b) Last year's income must have been

   $$m = 8000 \cdot 200 + 45000 \cdot 25 = 2725000$$

   This year's budget constraint is:

   $$13000 \cdot 205 + 45000 \cdot d^* = 2725000 \Rightarrow d^* = 1.3$$

   (c) The graph is:




(d) Hyundais are an inferior good because their Engel curve slopes down, and they are a Giffen good because the Marshallian demand curve slopes up due to the very strong income effect of a price change.