ECON 301, Professor Hogendorn

Problem Set 1

1. *Albums.* In 2012, sales of digital music albums were 118 (million) and sales of vinyl albums were 4.5. In 2013, sales of digital albums were the same at 118 and sales of vinyl albums were 6.

   Let's assume it's the "representative consumer" who buys these albums, so they're all in one indifference curve / budget line diagram. Assume that in both years the consumer makes a utility-maximizing decision.

   Consider each of the following changes in isolation. Are both of them consistent with the utility maximizing model and the consumption pattern described above? Illustrate on two separate diagrams with digital on the vertical axis and vinyl on the horizontal.

   (a) Price of vinyl declined and price of digital went up.

   (b) Price of vinyl declined and price of digital stayed the same.

2. *DanBrown.* Amazon sells the bestselling novel *Inferno* in different formats. Rounding the prices to the nearest dollar, the e-book edition on the Amazon Kindle costs $15 and the hardcover print edition costs $18.

   There are two types of consumers, affectionately called Inkies and Pixlees. Each type of consumer has $20 of income that they may allocate between the two types of books. (Don't worry about getting answers with fractional books.)

   (a) Write down the budget constraint for buying k units of the Kindle edition (horizontal axis) versus h units of the hard-

cover edition (vertical axis). Label the vertical and horizontal intercepts.

(b) Use the total differential to find the budget constraint's slope.

(c) Let Pixlees have utility function:

$$u(k,h) = k^{0.9}h^{0.1}$$

What is a Pixlee's marginal rate of substitution? What is their utility maximizing consumption of k and h?

(d) Inkies have a quasilinear utility:

$$u(k,h) = (k+1)^{0.5} + h$$

What do their indifference curves look like? Hint: what is the MRS when $k = 0$?

(e) How many books of each type do Inkies buy when they maximize utility?

3. *Martini2*. The Martini is a famous cocktail that is properly made with gin and vermouth. (Vodka martinis are a horrible travesty from the 1960s and 70s.)

Let the price of gin be \$1 per ounce and the price of vermouth be \$0.40 per ounce. Let G be ounces of gin and V be ounces of vermouth.

Do everything in this problem with G on the horiztonal axis and V on the vertical axis.

(a) Suppose you maximize utility function

$$u(G,V) = G^{0.9}V^{0.1}$$

subject to a budget constraint based on the above prices and some income m which is unknown. Use the MRS to show what ratio of gin to vermouth you use to make a martini.

(b) Suppose instead you maximize utility function

$$u(G,V) = \left(\frac{G}{25} - 1\right)^{0.5} + V$$

subject to the same budget constraint as above (again with unknown m). Use the MRS to determine how much gin you use in a martini. Why can't you determine the ratio of gin to vermouth using just the MRS?

(c) Find the ratio of G to V in part (b) as a function of m.

Review Problems Only, Not to Turn In:

4. *MRS.* Let an individual have the utility function

$$u(x,y) = x^{1/3}y^{2/3}$$

Use the standard space with x on the horizontal axis and y on the vertical axis for the following:

(a) Compute the marginal rate of substitution when $x = y = 8$. Draw on a graph.

(b) Compute the marginal rate of substitution when $x = 512$ and $y = 1$. Why is it different from part (b)? What does this explain about the person's preferences?

(c) Use the total differential to estimate the change in utility from starting at $x = y = 8$ and moving to $x = 512, y = 1$. (Use the point (8,8) to evaluate the partial derivatives.)

(d) The estimate from part (c) is much larger than the actual change in utility. Why? Since it's wrong, does it mean that differentials are not very useful in real life?

5. *Consistent.* When prices are $p_x = 1, p_y = 2$, a consumer demands $x = 1, y = 2$, and when prices are $p_x = 2, p_y = 1$, the consumer demands $x = 2, y = 1$. Is this behavior consistent with the model of

utility maximizing behavior? Draw an indifference curve / budget line diagram to illustrate your answer.

6. *BuyingXandY.* Suppose you have an income of $40 to spend on two commodities. Commodity X costs $10 per unit and commodity Y costs $5 per unit.

   (a) Write down your budget constraint. If you spent all your income on X, how much could you buy? If you spent all your income on Y, how much could you buy? Graph your budget constraint. What is its slope?

   (b) Suppose the price of X falls to $5 while everything else stays the same. Write down your new budget constraint. What is its slope? Graph your new budget constraint on the same graph as (a).

   (c) Suppose your income falls to $30, but the prices of X and Y remain at $5. Write down your new budget constraint. What is its slope? Graph your new budget constraint on the same graph as (a) and (b).

   (d) On your graph, shade in the area representing commodity bundles that you can afford with the budget in (c) but could not afford to buy with (a). Shade in the area representing commodity bundles that you could afford with the budget in (a) but cannot afford with the budget in (c).

7. *Jazz.* Suppose that there are two types of jazz music, smooth jazz and traditional jazz. The only jazz radio station in a city plays 10 minutes of smooth jazz for every 10 minutes of traditional jazz. Graph this point (10,10), and then analyze the preferences of the listeners. Assume all listeners have convex preferences, and there are two *equal-sized* groups of listeners, Group 1 and Group 2.

(a) At the (10,10) point, Group 1 listers have an MRS of 1 minute of traditional jazz per 4 minutes of smooth jazz. Graph the group 1 indifference curve through the (10,10) point.

(b) At the (10,10) point, Group 2 listers have an MRS of 8 minutes of traditional jazz per 1 minutes of smooth jazz. Graph the group 2 indifference curve through the (10,10) point.

(c) Suppose the radio station changes its format somewhat and plays 11 minutes of smooth jazz for each 9 minutes of traditional jazz. Which type of listener has moved further in space on the graph from its old indifference curve?

(d) Can we say which type of listener is better off and which worse off? Can we say whether the gains to one group more than offset the losses to the other?

Answers to Review Problems:

4. *MRS_a.* Let an individual have the utility function

$$u(x,y) = x^{1/3}y^{2/3}$$

Use the standard space with x on the horizontal axis and y on the vertical axis for the following:

(a) The MRS is the ratio of marginal utilities:

$$MRS = -\frac{MU_x}{MU_y} = \frac{\frac{1}{3}x^{-2/3}y^{2/3}}{\frac{2}{3}x^{1/3}y^{-1/3}} = \frac{1}{2}\frac{y}{x}$$

Evaluated at $x = y = 8$ this is $-\frac{1}{2}$. The graph should show a downward sloping line, flatter than a 45 degree line, and tangent to an indifference curve.

(b) Evaluated at $x = 512$, $y = 1$, the MRS is $-\frac{1}{1024}$. Relative to part (a), the ratio of x to y has increased a lot. Because the indifference curves are convex, this consumer tends to prefer

relatively balanced consumption bundles to extremes. Here the person is at an extreme, so a marginal increase in y brings much more utility than a marginal increase in x.

(c) The total differential is

$$du = \frac{1}{3}x^{-2/3}y^{2/3}dx + \frac{2}{3}x^{1/3}y^{-1/3}dy$$

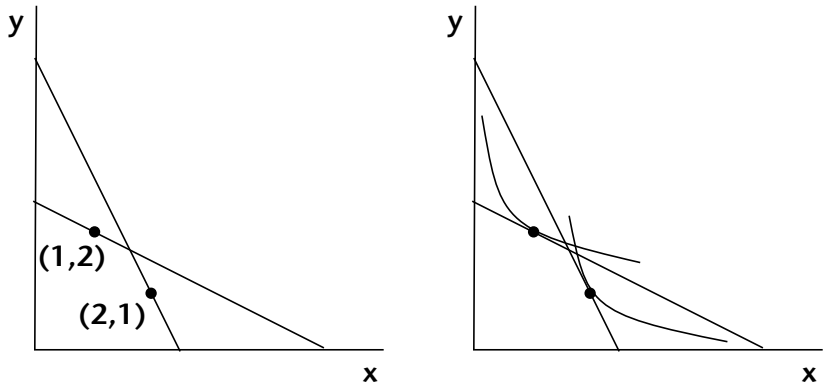
Moving from (8,8) to (512,1) gives $dx = 504$ and $dy = -7$. Evaluating from the starting point gives

$$\frac{1}{3} \times 1 \times 504 + \frac{2}{3} \times 1 \times -7 = 163.33$$

(d) If you actually plug (512,1) into the utility function, you'll discover that it gives a utility of 8, exactly the same as (8,8). So in fact, this is a movement along an indifference curve. But it's a very large movement. Since the differential is based on tangent lines, it linearly extrapolates the utility change, and the very large movement of x dominates. If you actually account for the curvature, the marginal utility of x diminishes a lot, and that of y increases, keeping the utility constant.

This does indicate that in real life, one must be careful using any calculus result, because it is only as accurate as a tangent. It is important not to apply calculus results over very large changes, unless the functions really are linear. On the other hand, as long as changes are small, or if you are only interested in the directions of the changes, differentials can be a very powerful way of determining complex relationships.

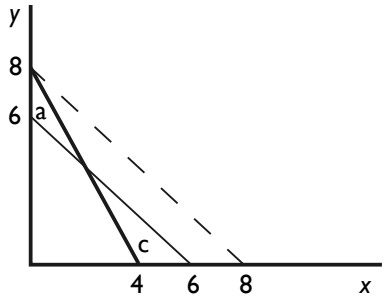
5. *Consistent_a.* First, just draw the budget lines and label the points.



Now if it is really true that both of these points are represent utility maximums, it must be that both are tangent to indifference curves. But this means the indifference curves must cross, since they are tangent to lines of different slopes. If indifference curves cross, then preferences are not monotonic – from some points, more of both goods would lower utility.

6. *BuyingXandY_a*

   (a) $10X + 5Y = 40$. If you spent all your income on X, you could buy 4. If you spent all your income on Y, you could buy 8. The slope is $-8/4 = -2$. This is represented by the dark budget line in the following graph.



   (b) $5X + 5Y = 40$. $Y = 8 - X$. Slope -1. Dashed line in graph.

(c) $5X+5Y=30$. $Y=6-X$. Slope -1. Narrow solid line in graph.

(d) Can afford area c with budget (c) but not (a). Can afford area a with budget (a) but not (c).

7. *Jazz_a.*

(a) A nice convex indifference curve.

(b) The curves cross. Group 2's is steeper.

(c) If a group had an MRS of -1, then that group would be indifferent to this movement. Both groups have MRSs that differ from -1: group 1's slope is 4 times less and group 2's slope is 8 times more. Thus Group 2's slope differs more and it will therefore move further in space from its old indifference curve (at least using the calculus approximation that is inherent in MRS).

(d) Group 1 is below its old indifference curve and is therefore worse off. Group 2 is above its old indifference curve and therefore better off. We can't compare utility measures between people, so we can't say whether the gains more than offset the losses.