ECON 301, Professor Hogendorn

Problem Set 4

1. *Snowboards.* There has been a recent collapse of interest in snowboarding. This is because of advances in ski technology that make skiing more fun for many people.

   (a) Draw a graph of the supply and demand of snowboards. Show what happens as a result of the decline in peoples' interest in snowboards. Label the old and new equilibrium quantities and prices.

   (b) Is there deadweight loss associated with the change in part (a)? Explain why or why not, and if there is, show it in the diagram.

   (c) Now let's be more specific. Snowboard market demand is $x(p) = a - p$ and snowboard market supply is $s(p) = 3p$. Let p^* be the equilibrium price. Find an expression for $\frac{dp^*}{da}$. Prove whether $\frac{dp^*}{da}$ is positive or negative.

   (d) And now a slightly harder version. Let's say we don't actually know the specific functional forms of demand. All we know is that there is some demand function $x(p, a)$ that is decreasing in p and increasing in a. And we know there is some supply function $s(p)$. And of course we know that supply equals demand. Using the total differential, find an expression for $\frac{dp^*}{da}$. Try to write the expression in elasticity form (but it won't be as neat as some of our other examples, you'll still have some extraneous stuff). Prove that the expression is positive.

   (e) Bonus +1 point, don't play around with this unless you have extra time. In (d), you could find an expression for the elasticity of p with respect to a. If you do that, you can make the entire expression a relationship between various elasticities.

2. *EducatedMothers.* A strong finding by development economists around the world is that when women are better educated, not only does their own standard of living rise, but so does their children's. The effect is largely due to the education of the mother herself, but also the average educational level of women in the community makes a difference. Suppose for example that a typical woman's utility function is $u_w(x, e) = x^{3/4}e^{1/4}$ where e is her own educational level and that a typical child's utility function is $u_c(e, E) = 12e^{1/16}E^{3/16}$ where e is the child's mother's education level and E is the average educational level of other women. Let there be one thousand women in the community. Note the child just takes x, e, and E as given.

   (a) Is there a positive externality in consumption of e? How does it operate? Do you expect that this externality will be internalized in any way? Intuitively (no math), what is the difference between the free-market e and the socially optimal levels?

   (b) Suppose the price of x is $p_x = 1$ and the price of e is p_e. What is the woman's MRS in (x, e) space? If the woman's income is 2,000, what is her private demand curve for education?

   (c) Suppose a social planner cares equally about women and children and that each woman has exactly three children. What is a social planner's MRS in (x, e) space? If the the a typical woman's income is 2,000, what is the social demand curve for education? What Pigouvian subsidy would correct the externality?

3. *Biofuel.* In the United States, the Federal Government gives has periodically given "blenders' credits" to fuel companies that blend

biofuels into their pretroleum-based gasoline or diesel. The blenders' credit is a subsidy that goes to buyers of biofuel, so within the biofuel market it should be viewed as a per-unit subsidy to consumers. To model this, let demand and supply be

$$x(p) = a - b(p - g) \qquad s(p) = \alpha p$$

where a, b, and α are parameters, p is price in cents per gallon, and g represents the blenders' credit.

(a) Show demand and supply curves (with and without the subdidy) and the deadweight loss on a graph.

(b) Find a formula for how much the government pays out in equilibrium (call this R). Use the derivative dR/dg to discuss how the government payout changes as g changes.

(c) (*Bonus + 1 point.*) What is the *elasticity* of the government payout with respect to the subsidy?

Review Problems, not to turn in:

4. *Laffer.* The "Laffer Curve" first became famous in the Reagan administration. It shows that when taxes are high enough, raising taxes can actually *reduce* tax revenue. While the theory is sound, its application to the U.S. income tax did not raise revenue under Reagan or either Bush.

   Suppose there is a tax t so that consumer pay price p and sellers receive price $p - t$. Demand is $x(p)$, and supply is $s(p - t)$.

   (a) Graph the tax and show the deadweight loss. Explain in words the deadweight loss.

   (b) What is the change in equilibrium price when the tax changes? I.e., what is $\frac{dp}{dt}$? Express this in terms of elasticities as much as possible.

(c) Now since tax revenue is $TR = tx(p)$, find $\frac{dTR}{dt}$. Put it in elasticity form as much as possible.

(d) Is it possible that raising the tax could reduce the revenue the government receives? Prove or disprove by signing the derivative from (c).

(e) Explain in words the logic of part (d). Make sure you use the word "elasticity" in your answer.

5. *Thornton.* Suppose the mayor of Middletown proposes a new tax on restaurant meals to finance Main Street improvements. Restaurant meals are elastically supplied at $s(p_s) = -5600 + 400p_s$. The tax will be a per unit tax, so the price restauranteurs receive is p_s and the price diners must pay is $p = p_s + t$. Demand for restaurant meals is $x(p) = 500 - 3p$.

   (a) Show the equilibrium price and quantity without the tax are \$15.14 and 456 respectively. Find the demand and supply elasticities at this equilibrium, and explain (in words) who will pay the tax, producers or consumers?

   (b) Show that the change in p_s when there is a change in the tax is 0.00744. Use the total derivative of the equilibrium condition.

   (c) Find a formula for $p_s(t)$, the equilibrium producer price given a tax of t. Then find formulas for $S(p_s(t))$, government revenue, and for deadweight loss as functions of t. The changes in government revenue and deadweight loss are respectively:

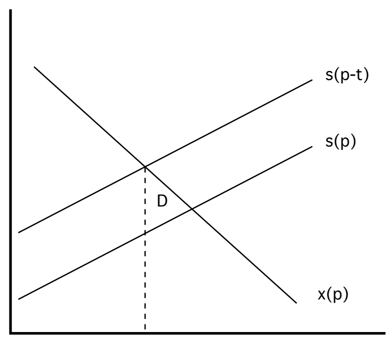
$$\frac{dR}{dt} = 456 - 5.96t \qquad \frac{dDWL}{dt} = 2.98t$$

   (d) Is it possible for the mayor to get in a situation where he or she cannot raise enough tax revenue to fund the improvements without causing more deadweight loss than the gains to Middletown from having the improvements? Explain with reference to the above formulas and to the elasticities of supply and demand.

Answer to Review Problems:

4. *Laffer_a.*

   (a) The graph is:



The DWL, labeled D, is the consumer plus producer surplus that is lost when the tax reduces the quantity consumed below the original equilibrium.

   (b) We know that supply must equal demand always:

$$x(p) = s(p - t)$$

Now we totally differentiate and rearrange the above:

$$\frac{\partial x}{\partial p} dp = \frac{\partial s}{\partial p} dp - \frac{\partial s}{\partial p} dt$$

(Note that the function s() only has one argument, therefore whether the p or the t in that one argument changes, the relevant change in s can still be denoted $\partial s / \partial p$.)

Now we solve for $\frac{dp}{dt}$:

$$\begin{aligned}\left(\frac{\partial x}{\partial p} - \frac{\partial s}{\partial p}\right) dp &= -\frac{\partial s}{\partial p} dt \\ \frac{dp}{dt} &= -\frac{\frac{\partial s}{\partial p}}{\frac{\partial x}{\partial p} - \frac{\partial s}{\partial p}}\end{aligned}$$

Now multiply the right hand side by (p/x)/(p/x) where x is the equilibrium quantity and is also equal to *s*:

$$\frac{dp}{dt} = -\frac{\epsilon_S}{\epsilon_D - \epsilon_S} = \frac{\epsilon_S}{\epsilon_S - \epsilon_D}$$

(c)

$$\begin{aligned}TR &= tx(p) \\ \frac{dTR}{dt} &= t\frac{\partial x}{\partial p}\frac{dp}{dt} + x(p) \\ &= t\frac{\partial x}{\partial p}\frac{\epsilon_S}{\epsilon_S - \epsilon_D} + x(p) \\ &= x(p)\left(t\frac{\partial x}{\partial p}\frac{1}{x(p)}\frac{\epsilon_S}{\epsilon_S - \epsilon_D} + 1\right) \\ &= \frac{x(p)}{p}\left(t\frac{\epsilon_D \epsilon_S}{\epsilon_S - \epsilon_D} + p\right)\end{aligned}$$

(d) For $\frac{dTR}{dt} < 0$, we can simplify the above further:

$$\begin{aligned}
\frac{dTR}{dt} &< 0 \\
\frac{x(p)}{p}\left(t\frac{\epsilon_D\epsilon_S}{\epsilon_S-\epsilon_D}+p\right) &< 0 \\
t\frac{\epsilon_D\epsilon_S}{\epsilon_S-\epsilon_D}+p &< 0 \\
\frac{\epsilon_D\epsilon_S}{\epsilon_S-\epsilon_D} &< -\frac{p}{t} \\
-\frac{t}{p} &< \frac{\epsilon_S-\epsilon_D}{\epsilon_D\epsilon_S} \\
\frac{t}{p} &> -\left(\frac{1}{\epsilon_D}-\frac{1}{\epsilon_S}\right) \\
\frac{t}{p} &> \frac{1}{|\epsilon_D|}+\frac{1}{\epsilon_S}
\end{aligned}$$

Sure, this could happen. You would need demand and supply to be relatively elastic and the tax rate t/p to be relatively high.

5. *Thornton_a.*

(a) Setting demand equal to supply gives:

$$500-3p_s = -5600+400p_s \Rightarrow p_s = 15.14, \quad s(15.14) = 456$$

The elasticities are $\epsilon = -0.1$ and $\epsilon_s = 13.28$.

(b) The equilibrium condition is

$$500-3(p_s+t) = -5600+400p_s$$

The total derivative is then

$$-3\left(\frac{dp_s}{dt}+1\right) = 400\frac{dp_s}{dt}$$

We can solve this for

$$\frac{dp_s}{dt} = -0.00744$$

(c)

$$
\begin{aligned}
&500 - 3(p_s + t) = -5600 + 400p_s \\
&6100 - 3t = 403p_s \\
&p_s = 15.14 - \frac{3}{403}t \\
&S(p_s) = -5600 + 400(15.14 - \frac{3}{403}t) = 456 - 2.98t
\end{aligned}
$$

$$
\begin{aligned}
\frac{dR}{dt} &= \frac{dtS(p_s)}{dt} \\
&= \frac{d456t - 2.98t^2}{dt} = 456 - 5.96t
\end{aligned}
$$

$$
\begin{aligned}
\frac{dDWL}{dt} &= \frac{d\frac{1}{2}(456 - s(p_s))t}{dt} \\
&= \frac{d\frac{1}{2}(456 - 456 + 2.98t)t}{dt} \\
&= \frac{d1.49t^2}{dt} = 2.98t
\end{aligned}
$$

(d) We know supply is very elastic and demand very inelastic. That means that adding a tax will basically increase the price to consumers a lot. As the tax increases, the marginal government revenue added goes down while the deadweight loss rises. Eventually, you reach a tax such that

$$\frac{dR}{dt} = 456 - 5.96t = 2.98t = \frac{dDWL}{dt} \Rightarrow t = 51$$

So, once the tax reaches 51, each marginal increase in tax causes more DWL than it does tax revenue.