

ECON 301, Professor Hogendorn

Problem Set 7

1. *Normal*. Find the Nash equilibrium(a) in this normal form game:

	L	C	R
T	(2,2)	(5,0)	(1,1)
M	(0,5)	(4,4)	(1,1)
B	(1,1)	(1,1)	(2,2)

2. *Tractors*. Two American companies, Case and John Deere, have decided to introduce their tractors in either the Polish market or the Hungarian market. Neither company has sufficient resources to enter both markets.

If they both enter the Polish market, they both expect profits of \$1 million. If they both enter the Hungarian market, they both expect profits of \$1.5 million.

If Case enters the Polish market and John Deere enters the Hungarian market, then Case expects profits of \$3 million and John Deere expects profits of \$4 million.

If Case enters the Hungarian market and John Deere enters the Polish market, then Case expects profits of \$5 million and John Deere expects profits of \$3 million.

There is a single consulting firm with special expertise that will enable either Case or John Deere to move first. The firm will offer its services to the highest bidder.

Use a normal form game to describe the most likely outcome.

- (a) Case outbids John Deere for the consultant's services. Case enters the Polish market first and then John Deere enters the Hungarian market.
  - (b) Case outbids John Deere for the consultant's services. Case enters the Hungarian market first and then John Deere enters the Polish market.
  - (c) John Deere outbids Case for the consultant's services. John Deere enters the Polish market first and then Case enters the Hungarian market.
  - (d) John Deere outbids Case for the consultant's services. John Deere enters the Hungarian market first and then Case enters the Polish market.
3. *Coke*. Suppose that all around the world, there are small towns in which the price elasticity of demand for Coca-cola is constant at -1.2. Each of these towns is served by a monopoly Coke distributor. However, the technology for distributing Coke varies widely: huge bottling plants and 18-wheeler truck delivery in the USA, local bottlers and van delivery in Japan, delivery by pack mule to isolated parts of Bolivia, etc.
- (a) What is the Lerner Index on Coke in these markets?
  - (b) Let the production function be  $f(K) = \beta K^2$ , where  $\beta$  is a parameter that varies from place to place, and let the price of capital be 20. How does the price of Coke vary with  $\beta$ ? (This is pretty tricky. Note that there is a constant elasticity demand, check review problem *Minus2*.)
4. *CheapB*. If firm A has a monopoly with demand curve  $p(Q) = 5 - 2Q$  and total cost curve  $TC(Q) = 2Q$ , it will maximize profits when it chooses  $Q_A = 3/4$  and  $p_A = 3.5$ . Its profit will be  $\pi_A = 1.125$ .

- (a) Firm B also has demand curve  $p(Q) = 5 - 2Q$ , but its total cost curve is  $TC(Q) = Q$ . Find the profit maximizing  $Q_B$ ,  $p_B$ , and  $\pi_B$  and draw on a graph.
- (b) Explain in words why firm B produces more than firm A.
- (c) Suppose firm A and firm B play a simultaneous move game to enter a market. If firm A enters and firm B does not enter, then firm A gets the monopoly payoff as above. Similarly if firm B enters and firm A does not. If neither firm enters, both get 0. If both firms enter, firm B will set price  $p = 3.4$  to undercut firm A. Firm A gets nothing and firm B gets the resulting profit based on a price of 3.4. Show the matrix form of this game. What is/are the Nash equilibrium(a)?
- (d) Suppose instead that the game is sequential. Firm A can choose enter or not enter, and then seeing this, Firm B chooses enter or not enter. The payoffs are the same as in part (c). What is the subgame perfection Nash equilibrium. Describe it in words, making sure to use the term “credible threat” or “incredible threat” as appropriate.

Review problem only, not to turn in:

- 5. *Varian27.1*. Varian, Chapter titled “Oligopoly,” Review Question #1.
- 6. *CreditCards*. Visa and Discover are considering the introduction of a new credit card service. Both firms have the same production function  $f(L, K) = L^8 K^3$ . Labor and capital both cost \$10 per unit.
  - (a) Assume  $K$  is fixed in the short run. Confirm that the short-run total cost curve is  $TC(y|K) = 10K + 10K^{-0.375}y^{1.25}$ .
  - (b) Suppose that Visa can move first and choose  $K = 17$  or  $K = 32$ , and Discover can see what it chose. Then Discover chooses

either  $K = 17$  or  $K = 32$ . Both firms compete using the cost curve from part (a). The way competition works is that the lower cost firm gets to sell 100 units at a price of 13 each. The higher cost firm exits the market – it gets no revenue but also has no costs, including no fixed cost of capital. In the event of a tie, both firms get to sell 50 units at a price of 13. Draw the extensive form of this game and fill in the payoffs.

- (c) What is the subgame perfect Nash equilibrium outcome?
- (d) Suppose Visa had an additional cost of 100 if it chose  $K = 32$ , but otherwise everything is the same. Does this change the subgame perfect Nash equilibrium? Does it suggest some type of contract that Visa might like to write with Discover?

### Answers to Review Problems:

5. *Varian27.1\_a*. First we need to set up the profit function for firm 1 and take the first order condition to get firm 1's best response function:

$$\max_{y_1} \pi_1 = (a - b(y_1 + y_2))y_1 - cy_1$$

Solving the first order condition gives:

$$\frac{\partial \pi_1}{\partial y_1} = (a - b(y_1 + y_2)) - by_1 - c = 0 \Rightarrow y_1 = \frac{a - by_2 - c}{2b}$$

The problem is identical for firm 2, so we also know that firm 2 will have a best response function

$$y_2 = \frac{a - by_1 - c}{2b}$$

A Cournot-Nash equilibrium is the quantity-pair such that both firms are playing their best responses simultaneously, so neither will want to deviate unilaterally. To find it, we just solve the best

response functions simultaneously:

$$\begin{aligned}
 y_1 &= \frac{a-c}{2b} - \frac{a-by_1-c}{4b} \\
 y_1 \left(1 - \frac{1}{4}\right) &= \frac{a-c}{4b} \\
 y_1 &= \frac{a-c}{3b}
 \end{aligned}$$

Since the problem is symmetric,  $y_2$  will be the same.

6. *CreditCards\_a.*

(a) From the production function,

$$y = K^{0.3}L^{0.8} \Rightarrow L^{0.8} = K^{-0.3}y$$

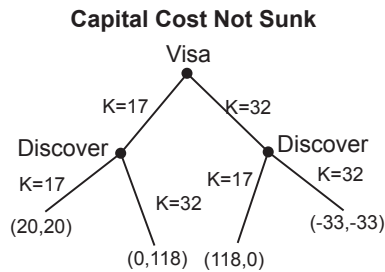
Thus, the short-run conditional factor demand for labor is

$$L(y|K) = K^{-0.375}y^{1.25}$$

With both the rental rate and the wage set to 10, the short-run total cost is

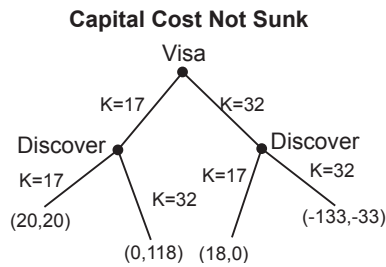
$$TC(y|K) = 10K + 10L(y|K) = 10K + 10K^{-0.375}y^{1.25}$$

(b) The extensive form game tree is:



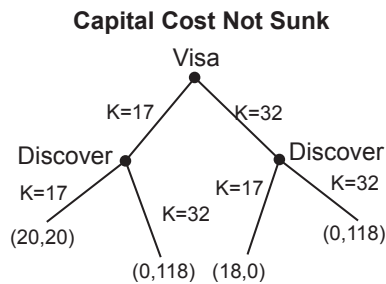
(c) The equilibrium of the left hand subgame is  $K=32$  and the equilibrium of the right hand subgame is  $K=17$ . By backward induction, Visa chooses  $K=32$ , preempting Discover. Discover does not have a credible threat to choose  $K=32$  in this case.

- (d) The simpler way to treat the change is to subtract 100 from Visa's payoffs when it chooses  $K=32$  and leave everything else unchanged:



This does not change the equilibrium, but it does make it sub-optimal: Visa gets 18 whereas it could get 20 from a cooperative contract where both choose  $K=17$ . Discover would also gain from the contract, going from 0 to 20.

A more subtle point is that the 100 cost to Visa may be counted in the short run total fixed cost that determines which firm get to sell 100 units. In that case, Discover now wins even in the case where both firms pick  $K=32$ :



Now the equilibrium of both subgames is for Discover to choose  $K=32$ , and the equilibrium of the whole game has Visa indifferent and choosing  $K=17$ . Visa would like to write the same contract discussed above, but its gain of 20 is not sufficient to compensate Discover for its loss of 98.