

ECON 301, Professor Hogendorn

Problem Set 8

1. *Catfood*. The game company PONOS has a monopoly over catfood, an in-game money in the game Battle Cats. Those who become addicted to Battle Cats very much want catfood, and there is no substitute. The current price of catfood is 3.3 (cents) per unit. The marginal cost of producing catfood is 0, which of course is the great advantage of in-game currencies and items.
 - (a) Assuming 3.3 cents is the monopoly profit-maximizing price, draw a linear demand curve, marginal revenue curve, marginal cost curve, profit maximizing price and and quantity. What rectangle represents PONOS's profit?
 - (b) At the price of 3.3 cents, is the demand for catfood elastic or inelastic? How do you know?
2. *KmartWalMart*. Suppose that Kmart and Wal-Mart both produce a composite output q which is some measure of floorspace and sales. Kmart's and Wal-Mart's cost curves are

$$c(q_K) = q_K \quad c(q_W) = 0.7q_W$$

The market demand for the composite good is $p(Q) = 500 - 4(q_K + q_W)$. The firms are Cournot competitors. What is the price and what are Kmart's and Wal-Mart's market shares and profits?

3. *AishaMrLee*. Aisha's utility function is

$$u(G, V) = G^{0.7} V^{0.3}$$

and Mr. Lee's utility function is

$$u(G, V) = G^{0.9} V^{0.1}$$

Aisha has 20 ounces of G and 10 ounces of V . Mr. Lee has 15 ounces of G and 15 ounces of V . This is all the G and V there is in the world, and there are no other people to trade with.

- (a) Calculate the MRS in (G, V) space for both consumers at the endowment point.
 - (b) Draw an Edgeworth box showing the endowment and indifference curves of the consumers. (The indifference curves do not have to be plotted to match the utility function perfectly.)
 - (c) Assume that Aisha and Mr. Lee can trade at a market price as price-takers. If we set G as the numeraire, what is the price of V ? What is the final allocation of G and V ?
 - (d) Show the trading in your diagram.
4. *MrLee*. Mr. Lee is an eccentric millionaire who made his money by manipulating the price of rice in Singapore. He now lives in Middletown, CT, where he purchased a defunct Bradlees department store and converted it to a house. In front of the house is a very large parking lot. Mr. Lee likes to consume large numbers of cars to fill up this parking lot (they can only be the latest model year, so he needs to buy a lot of new cars every year).

Last year the price of Hyundais was \$8,000 and the price of Mercedes was \$45,000. Mr. Lee bought 200 Hyundais and 25 Mercedes. These have now been towed away, and it is time to buy this year's cars. Unfortunately, the price of Hyundais has risen to \$13,000 this year.

The slope of Mr. Lee's Slutsky compensated demand function for Hyundais is -0.001 (i.e. one less Hyundai for each \$1,000 increase in price). The slope of his Engel curve for Hyundais is -0.00001 (i.e. one less Hyundai for each \$100,000 increase in income).

- (a) Using the Slutsky equation, what is the slope of Mr. Lee's Marshallian demand for Hyundais? How many does he buy this year (assuming the linear estimate of slope can be used)?
- (b) Assuming Mr. Lee's income did not change and he spends it all on Hyundais and Mercedes, how many Mercedes does he buy this year?
- (c) Graph Mr. Lee's consumption decisions in the two years using budget lines and indifference curves.
- (d) Which ones of the following describe Hyundais: normal good, inferior good, Giffen good?

Review problems only, not to turn in:

- 5. *Varian 27.1*. Varian, Chapter "Oligopoly," Review Question #1.
- 6. *Sopranos*. There are two goods, numeraire x and cooking c . The price of numeraire is always 1 throughout this problem, and the price of cooking is p_c .

Mrs. Soprano and Mrs. Bucco both have the same utility function:

$$u(x, c) = x^{0.8} c^{0.2}$$

Mrs. Soprano's endowment is $(\omega_{Sx}, \omega_{Sc}) = (100, 10)$. Mrs. Bucco's endowment is $(\omega_{Bx}, \omega_{Bc}) = (10, 10)$.

With this utility function and these endowments, the demand functions for numeraire for Mrs. Soprano and Mrs. Bucco are

$$x_S = 0.8 \frac{100 + 10p_c}{1} \quad x_B = 0.8 \frac{10 + 10p_c}{1}$$

- (a) If the two women can trade in an Edgeworth Box, what will be the final allocation and what will be the price of cooking?

(b) Suppose that the "powers that be" decide that this final allocation is not all right. They want the final allocation to be $(x_B, c_B) = (66, 12)$. Note that $(66, 12)$ IS on the contract curve. What lump sum taxes and subsidies on the numeraire are necessary to make this happen? Illustrate with an Edgeworth Box diagram.

7. *CreditCards*. Visa and Discover are considering the introduction of a new credit card service. Both firms have the same production function $f(L, K) = L^8 K^3$. Labor and capital both cost \$10 per unit.

(a) Assume K is fixed in the short run. Confirm that the short-run total cost curve is $TC(y|K) = 10K + 10K^{-0.375} y^{1.25}$.

(b) Suppose that Visa can move first and choose $K = 17$ or $K = 32$, and Discover can see what it chose. Then Discover chooses either $K = 17$ or $K = 32$. Both firms then compete using the cost curve from part (a). The way competition works is that the lower cost firm gets to sell 100 units at a price of 13 each. The higher cost firm exits the market – it gets no revenue but also has no costs, including no fixed cost of capital. In the event of a tie, both firms get to sell 50 units at a price of 13. Draw the extensive form of this game and fill in the payoffs.

(c) What is the subgame perfect Nash equilibrium outcome?

(d) Suppose Visa had an additional cost of 100 if it chose $K = 32$, but otherwise everything is the same. Does this change the subgame perfect Nash equilibrium? Does it suggest some type of contract that Visa might like to write with Discover?

8. *Pate*. There are two goods, beef (B) and goose liver pate (G). The typical French person has an endowment of $\omega_B = 50, \omega_G = 50$ and a utility function $U(B, G) = B^{0.3} G^{0.7}$. The typical American has an endowment of $\omega_B = 70, \omega_G = 30$ and a utility function $U(B, G) =$

$B^{0.8}$. Note that the typical American simply does not receive utility from the pate.

- (a) What is the typical French and American MRS in (B,G) space at the endowment points?
 - (b) Draw an Edgeworth box and show indifference curves for each type of consumer. Show the core and the contract curve.
9. *Pareto*. Is it possible to have a Pareto efficient allocation where someone is worse off than he is at an allocation that is not Pareto efficient? Illustrate with an Edgeworth Box.
10. *RichAndPoor*. A very rich person and a very poor person are going to trade in an Edgeworth box. The rich person is named Ms. 1 and her origin is the lower left corner. The poor person is named Mr. 2 and his origin is the upper right hand corner. The two people will trade good y (on the vertical axis) and good x (on the horizontal axis). Ms. 1 has the entire endowment of good x , and there is a lot of that good. Mr. 2 has the entire endowment of good y , but there is not that much of it. Both people's indifference curves indicate that good y doesn't bring very much utility compared to good x .
- (a) Draw the Edgeworth box, showing the endowment point, indifference curves, and the contract curve. What is the Walrasian equilibrium? Is it efficient?
 - (b) Suppose the government values equality and wants the final outcome of trading to be the allocation approximately in the center of the box. Show a government price control that forces the center point to be in the budget sets of both consumers. How does this change the Walrasian equilibrium? Is "equality" achieved? Is this solution efficient.
 - (c) Can the government use the Second Fundamental Theorem of Welfare Economics to improve on part (b)?

11. *BigMacs*. You buy a lot of Big Macs. You are also on your town's zoning board, and McDonald's REALLY wants to build a new restaurant there. McDonald's raises the price of Big Macs from \$3 to \$4. Your demand for Big Macs is $x(p, m) = 0.01 \frac{m}{p^{1.5}}$. Your income m is \$50,000.

You complain about the price increase, and subtly hint that it could affect your zoning decision. In response, McDonald's sends a representative who will compensate you with coupons for free Big Macs (fractional coupons are allowed). Here are three possible ways to compensate you:

- (a) Calculate a Laspeyres price index, calculate the additional income you would need according to the price index, and divide that amount by \$4 to get the number of Big Mac coupons.
- (b) Use Slutsky income-compensated demand to calculate the substitution effect, and give that many Big Mac coupons.
- (c) Use Marshallian demand to calculate the change in demand, and give that many Big Mac coupons.

Answers to Review Problems:

5. *Varian27.1_a*. First we need to set up the profit function for firm 1 and take the first order condition to get firm 1's best response function:

$$\max_{y_1} \pi_1 = (a - b(y_1 + y_2))y_1 - cy_1$$

Solving the first order condition gives:

$$\frac{\partial \pi_1}{\partial y_1} = (a - b(y_1 + y_2)) - by_1 - c = 0 \Rightarrow y_1 = \frac{a - by_2 - c}{2b}$$

The problem is identical for firm 2, so we also know that firm 2 will have a best response function

$$y_2 = \frac{a - by_1 - c}{2b}$$

A Cournot-Nash equilibrium is the quantity-pair such that both firms are playing their best responses simultaneously, so neither will want to deviate unilaterally. To find it, we just solve the best response functions simultaneously:

$$\begin{aligned} y_1 &= \frac{a - c}{2b} - \frac{a - by_1 - c}{4b} \\ y_1 \left(1 - \frac{1}{4}\right) &= \frac{a - c}{4b} \\ y_1 &= \frac{a - c}{3b} \end{aligned}$$

Since the problem is symmetric, y_2 will be the same.

6. *Sopranos_a*.

- (a) There are 110 units of numeraire in the economy, so we need

$$x_S + x_B = 0.8 \frac{100 + 10p_c}{1} + 0.8 \frac{10 + 10p_c}{1} = 110$$

Solving this gives $p_c = 1.375$.

This means that $x_S = 91$, $x_B = 19$, $c_S = 16.55$, $c_B = 3.45$. That is, Mrs. Bucco sells some cooking to Mrs. Soprano in exchange for numeraire.

- (b) Because of Walras' Law, all we need is to consider the demands for x . Note that if we take some amount of numeraire t from Mrs. Soprano and give it to Mrs. Bucco, the two women's demand curves become

$$x_S = 0.8 \frac{100 - t + 10p_c}{1}$$

$$x_B = 0.8 \frac{10 + t + 10p_c}{1}$$

When we add these up and set equal to 110, the lump-sum transfer t just cancels out, so the price of cooking is still $p_c = 1.375$. Then all we have to do is make sure that Mrs. Bucco consumes $x_B = 66$ in the final allocation, and we're done. Thus:

$$x_B = 0.8 \frac{10 + t + 10 \cdot 1.375}{1} = 66$$

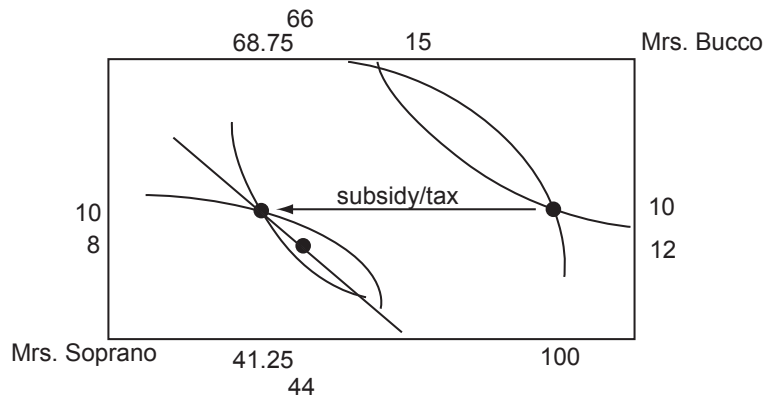
$$19 + 0.8t = 66$$

$$t = 58.75$$

To confirm this all works, consider that Mrs. Soprano must therefore consume the following amount of cooking:

$$c_S = 0.2 \frac{100 - 58.75 + 10 \cdot 1.375}{1.375} = 8$$

Since there are 20 units of cooking total and the goal was to have Mrs. Bucco consume 12 of them, Mrs. Soprano should consume 8, so this checks out. Note that the tax scheme reverses the trading: now Mrs. Soprano cooks for Mrs. Bucco!



7. *CreditCards_a.*

(a) From the production function,

$$y = K^{0.3}L^{0.8} \Rightarrow L^{0.8} = K^{-0.3}y$$

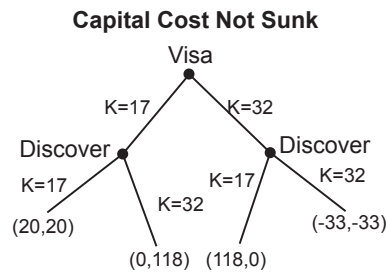
Thus, the short-run conditional factor demand for labor is

$$L(y|K) = K^{-0.375}y^{1.25}$$

With both the rental rate and the wage set to 10, the short-run total cost is

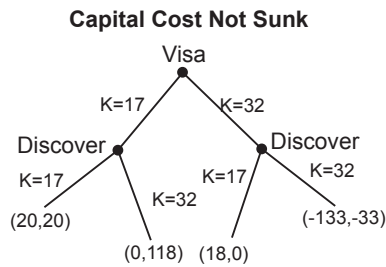
$$TC(y|K) = 10K + 10L(y|K) = 10K + 10K^{-0.375}y^{1.25}$$

(b) The extensive form game tree is:



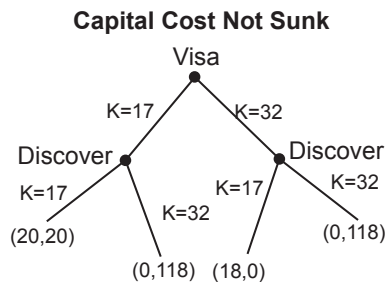
(c) The equilibrium of the left hand subgame is $K=32$ and the equilibrium of the right hand subgame is $K=17$. By backward induction, Visa chooses $K=32$, preempting Discover. Discover does not have a credible threat to choose $K=32$ in this case.

- (d) The simpler way to treat the change is to subtract 100 from Visa's payoffs when it chooses $K=32$ and leave everything else unchanged:



This does not change the equilibrium, but it does make it sub-optimal: Visa gets 18 whereas it could get 20 from a cooperative contract where both choose $K=17$. Discover would also gain from the contract, going from 0 to 20.

A more subtle point is that the 100 cost to Visa may be counted in the short run total fixed cost that determines which firm get to sell 100 units. In that case, Discover now wins even in the case where both firms pick $K=32$:



Now the equilibrium of both subgames is for Discover to choose $K=32$, and the equilibrium of the whole game has Visa indifferent and choosing $K=17$. Visa would like to write the same contract discussed above, but its gain of 20 is not sufficient to compensate Discover for its loss of 98.

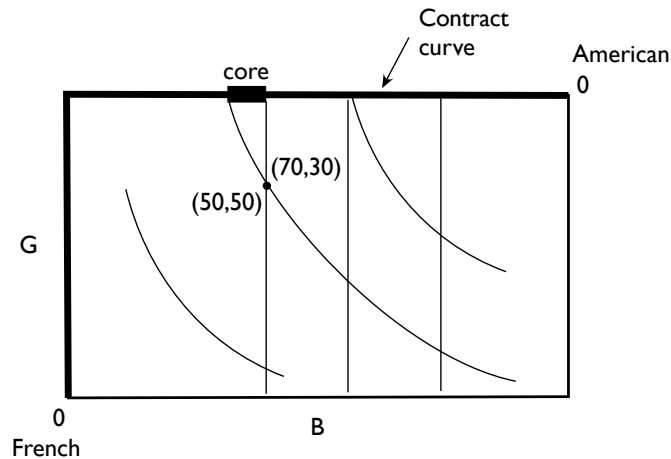
8. *Pate_a.*

(a) The French and American MRSs at the endowments are

$$\text{French: } -\frac{\frac{\partial U}{\partial B}}{\frac{\partial U}{\partial G}} = -\frac{0.3G^{0.7}B^{-0.7}}{0.7G^{-0.3}B^{0.3}} = -0.43\frac{G}{B} = -0.43$$

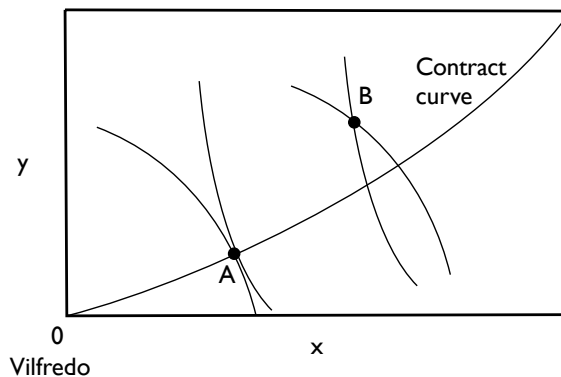
$$\text{American: } -\frac{\frac{\partial U}{\partial B}}{\frac{\partial U}{\partial G}} = -\frac{0.8B^{-0.2}}{0} = -\infty$$

(b) The only tangency between the indifference curves is when the French person has $B = 0$. Thus the whole left side of the diagram is part of the contract curve. In addition, all allocations where the American has $G = 0$ are Pareto efficient, and are therefore also on the contract curve. The core is the portion of the contract curve along which the both the French person and the American gain more utility than their endowments. In any trading, we would expect the American to trade away all her endowment of G .



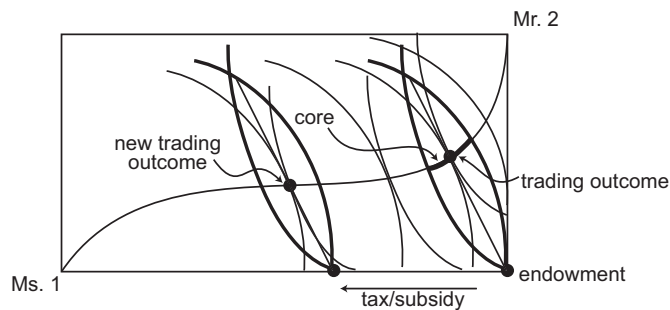
9. *Pareto_a*. Yes, Pareto efficiency says that it is not possible to make one person better off without making another person worse off. But that does not preclude making one person better off *and* making the the other worse off. For example, in the graph Vilfredo is

better off at point B than point A , even though B is not on the contract curve and not Pareto-efficient while A is.



10. *RichAndPoor_a.*

- (a) To show the relatively low value both people put on good y , we need a large MRS (steeply sloped indifference curves).
- (b) Although the center point would then be feasible from the point of view of the budget line, it would not be a Walrasian equilibrium. The indifference curves of the two consumers would be tangent at two different points along this budget line, so there would not be a market-clearing equilibrium. Some gains from trade would be lost, and the center point would not actually be achieved. In any case, unless the center point lies exactly on the contract curve, it is not efficient, since the consumers can Pareto-improve on it.
- (c) The second welfare theorem says that any point on the contract curve can be supported as a Walrasian equilibrium provided the consumers begin at the proper endowment point. In the graph, the government could lump-sum-redistribute good x from Ms. 1 to Mr. 2. Then the agents can trade at free-market prices and come as close to the center of the box as the contract curve will allow.



11. *BigMacs_a*.

(a) Laspeyres price index

$$\frac{p_{x2}x_1 + p_{y2}y_2}{p_{x1}x_1 + p_{y1}y_1} = \frac{4(96.23) + 49711.31}{3(96.23) + 49711.31} = 1.0019246$$

$$1.0019246 * 50000 = 50096.23$$

$$50096.23 - 50000 = 96.23$$

$$96.23/4 = 24.0575 \text{ coupons}$$

(b) Substitution effect:

We already saw that the Laspeyres price index adds 96.23 to income; this is the same as Slutsky income compensation.

$$x(4, 50096.23) - x(3, 50000) = 62.62 - 96.23 = -33.61$$

So this method gives 33.61 coupons.

(c) Marshallian demand:

$$x(4, 50000) - x(3, 50000) = 62.50 - 96.23 = -33.73$$

This method gives 33.73. It's a little more, but not surprisingly there is almost no income effect here because the share of income spent on Big Macs is still very small despite your gluttony.